P r e v i e w

Gifting Your Child:

Math and Literacy from Infancy

Kerman Bharucha

The full book titled above is a detailed account of my efforts at initiating a successful *very early childhood education* program for my grandson. The book then evolves into a book on basic math and science concepts.

This preview gives you: (a) the full table of contents as it appears in the book so that you can appreciate the logical sequence in which the teaching of math and literacy has been approached and what topics have been covered; (b) extracts of the text from selected sections of the book so you can appreciate my method of explanation of the various concepts covered in the book.

I hope that this preview will convince you that this book will show you the way to giving your child or grandchild an invaluable educational head-start in life.

I have stated in the book that in the early stages of the child's life, when the child has not as yet learned to read, you have to be the teacher. This implies that you must be knowledgeable in at least some areas of math. However, I have also emphasized that ***this book will give you that basic math knowledge, which you can then impart to your child***. Once your child gets to be reasonably proficient at reading, he or she can continue on with the book with little or no assistance from you.

As this preview is an extract from the book, the section-numbers (preceding the section-names) will not be consecutive, and the page-numbers shown in the table-of-contents will have no corelation here, so there are no page numbers in this document.

Thank you for taking the time to preview my book.

Kerman Bharucha
Webster, NY (USA)

ISBN 978-0-9960474-2-5

Education is not the learning of facts. It is the training of the mind to think

\- *Albert Einstein* -

This book is geared towards several audiences:

(A) The earlier sections of the book are for:

(1) prospective parents and new parents/grandparents who want to give their infant child a huge jump-start on his/her education;

(2) the parents of the pre-K child that needs a helping hand with math.

(B) The later sections of the book are for:

(1) the young student in elementary or middle school, who has reading skills but has difficulty with basic math;

(2) anyone who wants a quick review of basic math.

The table-of-contents is a helpful guide to the specific topics discussed in the book.

This book details my efforts at initiating a successful *very early childhood education* program for my grandson. After several consultations with my family, and out of deference to their wishes, I have decided to withdraw all references to his name from the book.

At this point, we feel it more prudent to safeguard my grandson's identity, so that we don't fall unwitting prey to the law of unintended consequences. Therefore, in this book, I will refer to my grandson only as "*Grandson*".

Gifting Your Child:
Math and Literacy from Infancy

Table of Contents

Level 4

Level 5

Level 6

Level 7

Level 8

Level 9

Level 10

Level 11

Level 12

Level 13

Level 14

Level 15

Level 16

Level 17

Level 18

Level 19

Level 20

Appendices

Introduction:

I have always had a love of learning, although I have to admit that I was not a very good student. I first graduated from college in 1969 with a Bachelor of Science degree from the University of Bombay (Mumbai) India, then went back to college 30 years later for a 4-year degree in Physician Assistant from Rochester Institute of Technology, Rochester NY.

(.....more detail is in the book.....)

By the time Grandson was just under 5 years old, I had managed to teach him an inordinate amount of complex math concepts (certainly complex for a 5-yr old), some of which would be considered by childhood-development experts to be quite impossible for such a tender mind to grasp. The majority of the concepts that I began teaching Grandson were based on math. However, there were also several non-math concepts that I worked on with him, as explained later in the book.

Here is a synopsis of what Grandson was taught by the age of 5:

1. Verbalize any number up to 36 digits long, giving you the decillion, nonillion, octillion, septillion, sextillion, quintillion, quadrillion, trillion, billion, million, thousand, and hundred in the number.
2. Write out the digits comprising any number up to 36 digits long, if you spelled out the number in a sentence. For example, if you wrote out: "One hundred and ninety-seven octillion, twelve quintillion, one thousand and eleven", he would read out the sentence to you, and then write out the number correct with all necessary zeros for fillers.
3. Count forward or backwards from any positive or negative number. He understood the concept of negative numbers.
4. Identify fractions as "proper" or "improper", and reason out which fraction was greater or lesser, and by how much.
5. Divide out any fraction correctly, regardless of the number of digits in the numerator; the denominator at that time was limited to a number from 1 through 12, because he had memorized his multiplication tables up to 12 by 12.
6. Correctly distribute given items based on simple ratios of 1:2, 2:3, 3:7, 4:7, etc. For example, if asked to divide 50 items in the ratio 2:3, he would correctly specify that one person would get 20 items and the other person would get 30 items.
7. Add, subtract and multiply two sets of numbers, regardless of the number of digits in each number.
8. Do negative subtractions directly, without repositioning numbers.
9. Understand that multiplication is merely a short-hand for repeated addition, and

division is merely a short-hand for repeated subtraction.
10. Answer questions of logic. Eg: "Zero is greater than all negative numbers." T/F
11. Draw the X and Y coordinate-axes, fill in the positive and negative numbers on both axes, number the quadrants, identify the "generic coordinate signs" in each quadrant, identify the coordinates of points marked on a grid, etc.
12. Recite and understand the definitions of "point", "line", "straight line", and "absolute value".
13. Draw a "compass" and correctly label up to 16 directions (north, north-northeast, north-east, east-northeast, east, etc).
14. Understand the concept of "60" as the limiting factor in clocks, and differentiate between counting down numbers in reverse and counting down a digital clock in reverse (where 3:00 would become 2:59), etc.
15. Read the time on an analog clock (with "hour" and "minute" hands).
16. Set up a chess-board correctly, and make basic moves for capturing pieces.
17. Read children's books quite well. etc.

The bottom line to my writing this book is that I want to detail the steps I took in teaching Grandson basic literacy, math and logical thinking and reasoning starting from an extremely young age, so that you can try using the same techniques to give your young child or grandchild an invaluable head-start in life. The path to success will, of course, take considerable time and effort on your part. If you are willing to provide the love, time and commitment, then this book will show you the way to achieving that goal.

If you're wondering why anyone would want to push so much information into such a young mind, my simple answer is that my primary aim was to encourage Grandson to learn to think logically, and to learn to reason. That was it. There was no other deeper hidden agenda. Albert Einstein has been quoted as saying: "Education is not the learning of facts. It is the training of the mind to think". I wanted to train my grandson to think, and the language of math, which is a short-hand notation for expressing simple and complex ideas succinctly, is a very good teacher of the fine art of logical thinking and reasoning.

(.....more detail is in the book.....)

My hope in writing this book is to inspire prospective parents, young parents and new grandparents to take a very early interest in their child's or grandchild's mental development. My experience with my children and with my grandson has taught me that infants have the ability to absorb, understand and process complex thoughts and concepts, and we do not give them enough credit on this score. There is plenty of medical and psychological literature out there to support this statement, if you want to do your own research. If you talk "baby-talk" to your infants, they will learn "baby-talk". If, on the other hand, you teach them numbers and other math and non-math concepts from an early age, they will learn to understand complex abstract thought.

A simple comparison to language-learning will bear out my point above. How difficult would it be for you, as an adult, to learn the Mandarin language today? You have to acknowledge that it would be extremely difficult! Yet, the child born in China learns to speak the language effortlessly, just through listening to his/her parents. ***It is the same principle at play between the infant brain and early childhood education imparted in the language of math and logical thinking***.

(.....more detail is in the book.....)

Starting off with Music and Number-Words: (*age of 1 week*)
Within one week of his birth, (and I mean that very literally - within 7 days) I started Grandson off on a regular dose of (a) music and (b) *"*number-**words***"* (his first exposure to numbers, through words). I was depending very heavily on the fact that infant minds are like sponges that absorb anything thrown their way. The infant's rate of learning far outpaces that of any adult because the infant brain receives a ton of unfamiliar sensory input every wakeful moment, and the infant now has to try and make sense of it all.

(a) Music: Experts have assured us - and it is my conviction - that music has a very positive impact on the infant brain, so I was sold on the idea that music had to play a big role in Grandson's life. For at least 2 hours every day (not always in one stretch), I would cradle Grandson in my arms, and he and I would listen to soft classical music that is orchestrated specially for the young.

(.....more detail is in the book.....)

(b) Number-**words**: I started verbalizing numbers to Grandson within a week of his birth. I would lie down next to him, then gently take hold of one of his hands. I would then physically trace my index finger along the length of each of his tiny fingers, while simultaneously whispering in his ear "one, two, three, four, five" successively for each finger. I would do this several times, with each hand. In effect, I was trying to reinforce the co-relation of tactile sensations in his fingers with the sounds of certain words. What I hoped to achieve through this repetitive exercise was to have Grandson mechanically associate his fingers in each hand with the words "one, two, three, four, five". Obviously, he had no idea what the words "one, two, three, four, five" meant, but at that time I had no doubt that repeating this exercise forged the words "one, two, three, four, five" indelibly in Grandson's infant mind, without his being aware of their true meaning. Over time, I put his hands together and physically traced all his fingers while verbalizing the numbers 1 through 10. Through this technique of tracing all ten fingers, I was hoping to get his mind to co-relate tactile sensations with different words.

This routine for music and for tracing number-words on his fingers went on periodically for about 3 months. There wasn't much else I could do at this time by way of his "education", because I was waiting for him to grow up just a little bit more.

(.....more detail is in the book.....)

<u>The Alphabet and Numbers</u>: (age around 3 months)
When Grandson was around 3 months old (an age when babies start using their hands and eyes in coordination and enjoy playing with people), I started him off on the recognition of the English alphabet and the numbers 0 through 9. Please note that I taught Grandson the alphabet and the basic numbers 0-9 in the same physical time-frame, from between 3 months of age to around 6 months or slightly beyond. While the following narrative details how I taught him to recognize letters, it summarizes my teaching him the basic numbers 0 through 9 because I used the same methods to teach him numbers as I had used with letters. To reiterate, he was taught the alphabet and the basic numbers 0-9 in the same time-frame of his life, between the ages of 3 months to 6 months.

<u>Teaching the alphabet (and numbers) to an infant</u>:
On the issue of teaching the alphabet to Grandson, I knew instinctively that it would be futile to begin with "A, B, C, D....". A 3-month old cannot relate directly to the letters "A, B, C, D". They mean nothing to a 3-month old. <u>You need to start off a 3-month old with the simplest letter in the alphabet, the big round "O"</u>.

I would sit in an inclined position on a sofa, and have Grandson sit comfortably in my lap, facing forward and resting his back against my chest. I would then take the index finger of his right hand, and trace a big "O" in the palm of my hand, while repeating some words to the effect "This is an 'Oh', round and round we go". I would repeat this little ditty with each "O" I traced on my palm. Through this exercise, I was hoping to reinforce in Grandson the co-relation of specific tactile sensations with the sound "Oh". I would then pick up any magazine lying around, and look for pages with uppercase "O"s. (If I could not find sufficient uppercase "O"s, I would write them in myself.) Then I would take Grandson's right index finger, and trace each "O" on the pages, while saying "Here is an 'Oh', round and round; here is another 'Oh', round and round; and here is another 'Oh', round and round". This was done with each trace. Within a very short time, we had reached the following milestone. I would pick up some magazine sheets with a few uppercase "O"s. I would then ask Grandson "Show me an 'O'", and he would move his little right index finger correctly to an "O". I would then say "Show me another 'O'", and he would show me the second "O" correctly, and so on for all the "O"s on the page. I cannot express how happy and wonderful it felt to realize that I had just taught a 3-month old how to recognize one letter of the alphabet. Through repetition, his infant brain had successfully co-related the round-symbol "O" (seeing) with the sound "Oh" (hearing) and with a specific circular-motion tactile sensation (touch).

(.....more detail is in the book.....)

The Alphabet (and numbers) continued: **Spatial visualization**:
Once Grandson had mastered the letters of the alphabet in uppercase, we took a major step forward. I wanted to reinforce his spatial visualization abilities. I would write down some uppercase letters on paper. Then I would point out to one of the letters, verbalize it, then slowly turn the page upside down so he could see the spatial rearrangement of the letter as it was being turned over, then I would verbalize it again as "upside down <letter>".

(.....more detail is in the book.....)

Time to start with "writing": (age around 7 months)
At around 7 months of age, Grandson could point out to me uppercase letters A-Z and numbers 0-9 quickly and comfortably from both perspectives, normal and upside-down. Up until this point in time, all I had used was his index finger to trace out the letters and the numbers in my palm, or to point them out to him on magazine sheets, or to have him point them out to me.

(.....more detail is in the book.....)

With Grandson sitting comfortably in my lap, and with the magnetic tablet in his lap, I would take his right-hand and wrap his small fingers as best as I could around the pen, then wrap my fingers over his while trying to emulate the writing posture that I normally use.

(.....more detail is in the book.....)

Pushing new material into Grandson: (age around 10 months)

(.....more detail is in the book.....)

(1) Progression to small words and the numbers 10-99:
At this point, our next logical progression was to put letters and numbers together in small packages.

(.....more detail is in the book.....)

(1-A) Progression to small words:
I started Grandson off in the study of word-series. This concept of "word-series" was simple. I wanted Grandson to start learning small words, but not just random words. They had to be logically related words. Just as we had practiced the letters of the alphabet in logical sequence based on their geometric shapes, we progressed to small words in their logical word-series.

(.....more detail is in the book.....)

(1-B) Progression to numbers 10-99:
Once Grandson could point out numbers to me from 0 through 9 accurately and quickly, I had to progress him to higher numbers. Continuing to use the hand-on-hand approach with the magnetic tablet instrument, I had Grandson practice tracing out, and listening to, numbers 10 and beyond. I verbalized each number out loud, for reinforcement. But I also verbally emphasized the "duality" of the numbers beyond 9. For example, while writing out some numbers like 11 and 12 and 13, I would say "one-and-one is eleven"; "one-and-two is twelve", "one-and-three is thirteen", etc., trying to emphasize to Grandson how two individual numbers generate a very different-sounding number when used together.

(.....more detail is in the book.....)

(2) Progression to lower-case letters:
This was something I was not particularly looking forward to, because I knew that I would be introducing a ton of confusion to my young man, and I knew that we would essentially be going back to the drawing board (the magnetic tablet, actually) and starting off on letters all over again, but it was something that just had to be done.

(.....more detail is in the book.....)

(3) Reading simple children's books to Grandson, with finger-tracing:
Only when Grandson was reasonably comfortable with pointing out letters to me in lower-case did we progress to our next logical goal, which was to start "reading". Obviously, at this point he was not talking yet, so I would be the one doing the reading and he would be listening. There are hundreds of excellent books out there for very young children, and I bought a few and borrowed a few from our local library.
Our reading routine was similar to the routines I had used in other contexts, that of Grandson sitting comfortably in my lap, and a small book opened up in his lap, then taking his right index finger and moving it across each word as I verbalized it aloud. My objective here was to get Grandson to visually recognize, and start to feel more comfortable with, the lower-case letters as he encountered them in printed words. There were times when I noticed that his eyes were on the colorful pictures on the page and were not following his finger, and I would gently tap his finger on the page

and bring his attention back to bear on the word at hand.

(.....more detail is in the book.....)

As of this time Grandson could not talk as yet, and I was content to do all the talking and have him do all the listening. I also realized at this time that I could not push any more information into Grandson until he had matured a bit more, *so for the next few months we continued in a holding pattern of concept repetition and reinforcement on everything we had learned up until then*. However, I was very happy in the thought that Grandson had picked up a great deal of information at a very young age, and I had absolutely no doubt that pumping all this information into him at such a tender age would serve him well in his memory development.

Over the next few months, somewhere between the ages of 26 to 30 months, Grandson learned to talk reasonably well.

(.....more detail is in the book.....)

(4) Increase in Grandson's words-vocabulary: When Grandson began to talk, our interaction became that much more interesting.

(.....more detail is in the book.....)

(5) Increase in Grandson's numbers-vocabulary:
Another big advantage of Grandson's talking and interacting with me was that it also allowed him to vocalize numbers. For example, now it was his turn to say to me "3 and 4 make thirty-four, 4 and 4 make forty-four, 5 and 4 make fifty-four", etc. Over time he became very efficient at recognizing and saying out numbers between 1 and 100. Here is an interesting side-clip.

(.....more detail is in the book.....)

(6) Skip-counting:
Our next logical step with numbers was having Grandson count forward by 2s, then by 5s, then by 10s.

(.....more detail is in the book.....)

(7) Reverse counting:
Our next logical milestone of course was having Grandson count backwards sequentially from any number up to 100.

(.....more detail is in the book.....)

(8) The limiting factor of "60" in clocks:
I was now ready to explain to Grandson a crucial concept, that of "60" as the limiting factor in watches and clocks. But before I could do that, I had to introduce Grandson to time-pieces in general.

(.....more detail is in the book.....)

(9) Reading an analog time-piece:
Obviously, once the concept of 60 seconds to a minute and 60 minutes to an hour was established, our next logical step was to ensure that Grandson could read an analog time-piece quickly. To do this, we had to again bring up "counting by 5s" around the face of a watch. The objective now was to associate the verbal counts of 5-10-15, etc., with the hours 1-2-3, etc., around the dial of the watch.

(.....more detail is in the book.....)

(10) Counting a clock down in reverse:
The next logical and crucial progression was to have Grandson count-down a clock in reverse, which of course is not the same as counting down numbers in reverse because of the limiting factor of 60.

(.....more detail is in the book.....)

(11) Understanding a directional compass: (long, but very necessary)
It was very important to me that Grandson should have some basic knowledge of "directions" as an important concept to understand, not just for the sake of knowing directions in general, but because the effort made here would also help him greatly with spatial visualization and memory enhancement. It's important that children practice spatial visualization (one of my personal convictions).

(.....more detail is in the book.....)

(12) Numbers 100 to 999:
The next logical step should be to have your child understand numbers between 100 and 999. Because the decimal system is so symmetrical and repetitious, your child should not have major difficulty in grasping numbers greater than 100. Write down the number 101, and have your child repeat after you "one hundred and one". Now write down 201, and ask your child: "If 1-0-1 is one hundred and one, what do you think 2-0-1 would be?" Hopefully, you will get the correct answer. If not, backtrack a bit and try again.

(.....more detail is in the book.....)

Stay on this path for as long as it takes your child to verbalize any 3-digit number without hesitation, because understanding all higher number concepts will depend on the child being able to verbalize 3-digit numbers rapidly.

(13) Units, tens, hundreds, and the place value of a digit:
These are very important concepts because they are a cornerstone to a clear understanding of much larger numbers, so take as much time as is needed for your child to understand the following concepts. By the time you are done with this section and the next one, your child will be able to verbalize numbers up to 36 digits long!

It is important that your child is able to readily verbalize 3-digit numbers. If this is not the case, take a temporary hiatus on this section, and review the previous sections.

(.....more detail is in the book.....)

A better way to drill home the concept of "the place value of a digit" is to use the same digit in a 3-digit number. For example, write down the number 555.

(.....more detail is in the book.....)

Again, your child must be able to freely verbalize 3-digit numbers, to proceed ahead with the recognition of larger numbers.

(14) Verbalizing hundreds, thousands, millions, billions, trillions.....:
Start on this section only if your child can effortlessly verbalize numbers up to 3-digits, and can identify the actual value of any digit in the number, depending on its position (place). If this milestone is achieved, we can now go ahead and take a giant leap forward in your child's understanding of numbers.

Write out any string of numbers as for example: 123456789234567890

Now start grouping them in 3s, from the right hand side. Your number should look something like this: 123 456 789 234 567 890

Tell your child that all numbers should be grouped in 3s from the right hand side, and it does not matter if the leftmost group has less than 3 digits.

Now we name the groups, starting from the right hand side. Taking as our example the same number above, the group-names (read them from right to left) are:

<<<<<-----<<<<<-----<<<<<-----<<<<<-----<<<<<-----<<<<<-----<<<<<--

123	456	789	234	567	890
quadrillion	trillion	billion	million	thousand	hundred

Therefore, the above number would be verbalized as "One hundred and twenty-three quadrillion, four hundred and fifty-six trillion, seven hundred and eighty-nine billion, two hundred and thirty four million, five hundred and sixty-seven thousand and eight hundred and ninety".

Note very carefully!   This point is crucial to understand!
In any group of 3 digits above, the rightmost digit is still "units", the middle one is still "tens" and the leftmost is still "hundreds". For example, in the above 567 (which is in the "thousand" group), the 567 by itself is verbalized as "five hundred and sixty-seven", but we add the qualifier "thousand" after that number, because the 567 is in the "thousand" group. It is not just 567 as a stand-alone 3-digit number, in which case it would only be "five hundred and sixty-seven" in value. This "567" brings "567 thousand" to the overall number.

In the above 789 (billion group), the 789 by itself is verbalized as "seven hundred and eighty-nine", but we add the qualifier "billion" after that number, because the 789 is in the "billion" group. It brings "789 billion" to the overall number.

As with any 3-digit number, the location of a digit in a number-group determines the actual value that digit will bring to the number-group. For example: in the "789 billion" group, the 7 brings a value of 700 billion to the overall number, the 8 brings a value of 80 billion to the overall number and the 9 brings a value of 9 billion to the overall number. Make sure your child understands this.

(.....more detail is in the book.....)

With some memorization, your child should be able to recite the group names rapidly from right to left (lowest to highest), and from left to right (highest to lowest). I made Grandson memorize 12 group-names as follows, forwards and backwards. They are written down in the text from highest (leftmost) to lowest (rightmost).

Decillion, Nonillion, Octillion, Septillion, Sextillion, Quintillion, Quadrillion, Trillion, Billion, Million, Thousand, Hundred.

Note that there are 12 groups, each with 3-digits in them, therefore with some practice and some memorization your child will be able to verbalize any number up to 36 digits long. Imagine the mental gyrations your child will be forced to go through to master this concept and memorize the 12 group-names from left to right and from right to left.

(.....more detail is in the book.....)

Examples:
1,234 <---- read R to L ----
The groups in 3s from right to left are "thousand / hundred", so this number is "one thousand / two hundred and thirty four".

12,345 <---- read R to L ----
The groups in 3s from right to left are "thousand / hundred", so this number is "twelve thousand / three hundred and forty-five".

(.....more detail is in the book.....)

And now for the big 36-digit enchilada.....

205,000,406,202,489,899,010,000,003,007,292,004

The groups in 3s from right to left are: (read both lines from R to L)
"decillion / nonillion / octillion / septillion / sextillion / quintillion / quadrillion / trillion / billion / million / thousand / hundred".

In the above number, the groups in 3s from right to left are all 12 group names given earlier, so this number is:

"two hundred and five decillion / no nonillion / four hundred and six octillion / two hundred and two septillion / four hundred and eighty-nine sextillion / eight hundred and ninety-nine quintillion / ten quadrillion / no trillion / three billion / seven million / two hundred and ninety-two thousand / and four".
(.....more detail is in the book.....)

(15) Writing out hundreds, thousands, millions, billions, trillions**.....**
At this point, let's take the following logical enhancement, and have your child write out large numbers. You spell out a large number in words, then have your child read the sentence, and write out the number. For example, you write out "Fourteen thousand and seven"; your child will read the sentence out loud, then write down "14,007" as the answer. Obviously, the child must know the group-names backwards and forwards, and the child must be able to read reasonably well before you can work on this section.

Presuming for now that your child knows the number-groups well and can read reasonably well, let's practice writing out large numbers.

Suppose the child has to write out "Eight trillion, seven thousand and four" as a number.

Step-1: Write down the number 8, and say it aloud as "8 trillion". So at this point, the number is simply: 8 (but in the child's mind it is 8 "trillion".)

Step-2: After the 8 "trillion", the next group should be "billion". But there is no "billion" mentioned in the number to be written out. So say "no billion" and put down 3-zeros, to represent the "billions" group. So at this point, the number is: 8 000

(.....more detail is in the book.....)

The final number will now be written out as: 8 000 000 007 004

If you work out the above number in groups of 3, you will have the groups mentioned in the number (read from R to L): trillion, billion, million, thousand, hundred.

(.....more detail is in the book.....)

Why did I work with Grandson on such large numbers? After all, there may not be enough grains of sand on Earth to come close to a decillion. However, as stated so many times before, I want to challenge Grandson's capacity to conceptualize, memorize, rationalize, spatialize, verbalize and any other *-ize* you can think of. I guess my daughter is right. I am a compulsive neurotic. But then think of the mental challenges I have put Grandson through.

(16) Defining "point", "line", "straight line":
We will take a small but crucial tangent here to explain the following very important definitions and concepts to your child. He needs to understand these concepts and mull them around in his head, and memorize these definitions.

(.....more detail is in the book.....)

(17) The all-important "number axis" and the true meaning of Numbers:
You are now at a very important cross-roads in your child's education. You have to impart to him the true meaning of numbers. Your child needs to be very comfortable with this concept, to be able to work on the many other nuances concerning numbers.

(.....more detail is in the book.....)

If your child has a very clear comprehension of the number-axis, we can now tackle the concepts of basic Addition and Subtraction, using the number-axis. "Real world" addition and subtraction is covered later in the appropriate sections.

(23) Addition for the real world:
As useful as the number-axis is for a clear understanding of the many concepts of numbers, it is not a practical tool for daily usage. In school, for example, your child will be asked to add numbers such as:

```
  15      23      85      409      125      600      909      1010
 + 5     +35     +29     +  9     +103     +202     + 54     + 987
 -----   -----   ----    -----    ------   ------   --------  ---------
```

(.....more detail is in the book.....)

(27) Subtraction for the real world:
As useful as the number-axis is for a clear understanding of the many concepts of numbers, it is not a practical tool for daily usage. In school, for example, your child will be asked to subtract numbers such as:

```
 15       23       85       409       125       600       909       1010
 -  5     -35      -29      -  9      -103      -202      - 54      - 987
-------  --------  --------  --------  ---------  -------  --------  --------
```

(.....more detail is in the book.....)

Before we start on the process of subtraction, emphasize the following to your child:

(1) Never proceed blindly with a subtraction process. First look at the problem carefully, and ask yourself if the result will be a negative number, before proceeding with the subtraction. For example:

(.....more detail is in the book.....)

The point being emphasized right now is that you should <u>never proceed blindly with a subtraction process</u>. First ask yourself if the result will be a negative number, before proceeding with the subtraction. If the answer is "yes, the result will be a negative number", then we either proceed with a "twist" (explained later), or use the direct process for working out that subtraction (covered in the next section).

(.....more detail is in the book.....)

(5) <u>Read this one carefully, and memorize it!</u>
There will be times when you will not be able to make a logical subtraction in a specific column. For example, 4 - 9. Logically, you cannot subtract a larger number from a smaller number. So you help the smaller number in that column by adding 10 to the smaller number. If you so "help" the smaller number in any column, you have to

(.....more detail is in the book.....)

(28) Subtraction leading to a negative answer:
Remind your child: Before attempting a subtraction, take a quick second look at the numbers given to you. If you are attempting to subtract a larger number from a smaller one, the answer will always be negative.

(.....more detail is in the book.....)

(29) Direct subtraction leading to a negative answer:
This concept provides very good mental exercise for your child. He will have to think outside the box on this one. Look at the following subtraction.

(.....more detail is in the book.....)

****************************** Tangent ******************************

For the subtraction process, I have followed the method that if you take "help" in a specific column, you must

(.....more detail is in the book.....)

However, it is possible that your child may be taught a slightly different method in his school. Let's call this "method 2". This method says that if you take "help" in a specific column, you must

(.....more detail is in the book.....)

*************************** End of Tangent ***************************

(32) The concept of "Absolute Value":
The "absolute value" of a number tells us <u>how far the number is away from the origin</u>.
Example: The "absolute value" of 9 is 9 because 9 is 9 units away from the origin.
The "absolute value" of -9 is 9 because -9 is 9 units away from the origin
Therefore: "absolute value" of 9 = "absolute value" of -9, because both 9 and -9 are 9 units from the origin. Simply put, the "absolute value" of a number is the number by itself, disregarding the <u>sign</u>.

(.....more detail is in the book.....)

(33) Multiplication:
As with the other concepts in math, this one is also crucial.

Explain to your child that "<u>multiplication is merely a short-hand for repeated addition</u>".

Say you want to add 7 to itself 10 times. You could express this as: 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7. And when you're done adding this up, you will hit the value 70. A much shorter and faster way to express "7 added to itself 10 times" would be: **7 x 10**. This is much shorter and faster than writing 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7.

To express "5 added to itself 12 times", you could say "5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5", or you could say 5 x 12. Clearly, the second way is so much shorter and faster.

(.....more detail is in the book.....)

(34) Multiplication for the real world:
In school, your child will have to deal with the following types of multiplication problems:

```
  4          27           2105          30297          13000
x 7        x  35        x    86        x 319          x    3
--------   ----------   ---------     ----------     -----------
```

Before we start on the process of multiplication, emphasize the following to your child:

(1) We will think of the lower-row number as the multiplier number, and the upper-row number as the number being multiplied (also called the "multiplicand"). In the above examples, the multipliers in each problem are 7, 35, 86, 319 and 3. In reality, of course, A x B = B x A, so if we "turn the numbers over", then what was the "multiplier" now becomes the "multiplicand", and vice-versa. So the terms "multiplier" and "multiplicand" are very relative. Don't lose sleep over them. I like to think of them all as "the multipliers", because they are multiplying with each other.

(.....more detail is in the book.....)

(39) The concept of a "matrix":
Now that your child has a basic understanding of multiplication, the next logical concept for your child to grasp is that of the "matrix". While the word "matrix" has many meanings in science and medicine, in basic math terms a matrix is a symmetrical arrangement of items in the form of a square or a rectangle. The items could be anything: cans, bottles, fruits, flower-pots, coins, people, boxes, etc. Our objective here is to get your child to understand some simple math associated with matrices (plural of "matrix").

(.....more detail is in the book.....)

(40) Squares, cubes, etc., and roots:
The topic of "squares" and "cubes" builds upon the concept of "multiplication", but refers to a number multiplying itself a few times. This topic also reminds you that the language of math is a short-hand notation for expressing simple and complex ideas succinctly.

(.....more detail is in the book.....)

Let's discuss squares, cubes, etc. first:
If I want to show "3 multiplied by itself 2 times", I can show it as 3 x 3 or as $\mathbf{3^2}$.
The small 2 (shown above and to the right of the number 3) is called the "index or the power or the exponent to which the 3 is raised", and implies that the 3 is multiplied by itself 2 times. The answer here, of course, is that 3x3 (or 3^2) = 9. Multiplying a number by itself 2 times is also called "squaring a number", and the result is called the "square" of the original number. For example: $5^2 = 25$. We verbalize this as: "5 squared is 25". Here, the 5 is "squared", and the 25 is "the square of 5".

If I want to show "4 multiplied by itself 3 times", I can show it as 4 x 4 x 4 or as $\mathbf{4^3}$.
Again, the index (3) written above and to the right of the number 4 tells us that the main number (4) is to be multiplied by itself 3 times. In this case: $4 \times 4 \times 4 = 4^3 = 64$.
Multiplying a number by itself 3 times is also called "cubing a number", and the result is called the "cube" of the original number.

For example: $5^3 = 5 \times 5 \times 5 = 125$ We verbalize this as: "5 cubed is 125".
Here, the 5 is "cubed", and the 125 is "the cube of 5".

(.....more detail is in the book.....)

(41) Division:
As with the other concepts in math, this one is also crucial.
Explain to your child that "division is a short-hand for repeated subtraction".

For example: What is 30 divided by (or divided into) 6 ?
(There has always been a controversy about the phrase "divided into". More on that later, under "Fractions".)

So: What is 30 divided by (or divided into) 6 ?
In other words: From 30, how many times can you remove (subtract) 6, until the 30 is all gone? One way to figure this out is to go through the detailed subtraction steps:
30-6=24 (*1 time*); now 24-6=18 (*2 times*); now 18-6=12 (*3 times*); now 12-6=6 (*4 times*); now 6-6=0 (*5 times*).

Note that we could subtract (take away, or remove) 6 from 30 a total of 5 times before the 30 is reduced to zero. Therefore, 30 divided by (or divided into) 6 is equal to 5.

(.....more detail is in the book.....)

There are two crucial points to understand about division involving a zero:

(.....more detail is in the book.....)

(42) Division for the real world:

(.....more detail is in the book.....)

(46) Distributing items based on a ratio:
If you feel that your child is reasonably comfortable with basic multiplication and division, we should now look at the concept of "distributing items based on a ratio". This is a very interesting and useful concept that your child needs to know. I had honed this concept into Grandson when he was very young.

Problem 1: You want to distribute 20 marbles between two kids, "in the ratio 2 to 3". How many marbles would each kid get ?

First, let us understand the meaning of the phrase "in the ratio 2 to 3".
A "ratio 2 to 3" means that if one person gets 2 of something, another person gets 3 of the same. Extending the concept, for every 2 of something given to person-A, you give 3 of the same thing to person-B. You continue this process until you are out of whatever it is that you are distributing.

"A ratio 5 to 7" means that for every 5 items you distribute one way, you are distributing 7 of the same items some other way.

Note: it does not have to involve a "person". You may want to distribute 20 pairs of socks between two drawers in the ratio 2 to 3, or you may want to distribute 54 packages between two trucks in the ratio of 4 to 5 etc., etc.

Note: "a ratio 2 to 3" is written as 2**:**3 (note the colon between the two numbers)
"a ratio 5 to 7" is written as 5**:**7 (note the colon between the two numbers) etc.

(.....more detail is in the book.....)

(48) Fractions - proper:
"Fractions" is one of those critical topics in which your child needs to have a very thorough grounding. You would be surprised to know how many adults out there have difficulty even with the most basic concepts involving fractions, never mind the

various nuances associated with fractions.

Check out the following numbers: $\frac{2}{3}$ $\frac{5}{7}$ $\frac{4}{9}$ $\frac{7}{11}$ $\frac{9}{17}$ $\frac{6}{13}$

These numbers are called "Fractions". I will explain very soon why they are called "fractions". For now, let's see the several ways of verbalizing these numbers.

Example 1: 2/3 can be verbalized as: "two upon three"; "two divided by three"; "two divided into three"; "two-thirds". But the best way by far to verbalize this number, so as to bring out its true meaning, is "two <u>out of</u> three".

Example 2: 5/7 can be verbalized as: "five upon seven"; "five divided by seven"; "five divided into seven"; "five-sevenths". But the best way by far to verbalize this number, so as to bring out its true meaning, is "five <u>out of</u> seven".

(.....more detail is in the book.....)

Now let's understand this clearly. If I have 3 items in hand, and I gave you 2 out of the 3 items, I have <u>not</u> given you all 3 available items. I have only given you <u>2</u> out of the 3 items. I have only given you a portion or a fragment of the total number of available items. This portion or fragment is called a <u>fraction</u>. And because I <u>can</u> physically give you 2 items out of 3 items, "2/3" is called a "proper fraction" (also called "real fraction", or "true fraction").

(.....more detail is in the book.....)

<u>(61) Mathematical expressions</u>:

Anything legitimate that you can express with numbers, letters, operations (addition, subtraction, multiplication, division), math-symbols (plus, minus, equals, square-root, cube-root, squares, cubes, etc), is a "mathematical expression". Examples are:

(1) 0 (or any other number) is a math expression;

(2) $(a + b) - (c - d) / 7$ is a math expression;

(3) $$\frac{(2 + 3x^2) * 4-\sqrt{(4a - 3b +c)}}{6 - x^3}$$ is a math expression, etc.

The only requirement is that the expression has to be valid. For example, showing a "division by zero" would not be a valid expression, or asking for the square-root (or any even-numbered root) of a negative number would not be a valid expression, etc. So, all the fractions, equations, additions, subtractions, multiplications, divisions, etc.

that we have seen so far, and which you will encounter further along in the book, are math expressions.

(.....more detail is in the book.....)

(63) Decimals:

I know that you have heard this *ad nauseam*, but this topic (like every other topic in this book) is also very important for your child to understand. He cannot escape his destiny with the decimal system.

The word "decimal" derives from the Greek word "deca", meaning 10. Our familiar number system is called the "decimal" number system because it is founded on a base (a foundation) of only 10 unique numbers (or symbols). Note the word "unique".

The unique numbers (or symbols) that form the base of our number-system are: "0,1,2,3,4,5,6,7,8,9". The numbers 0 through 9 (that makes 10) are the only 10 unique numbers (symbols) that make up our number system. Any value beyond 9 has to be shown as two or more of the 10 base unique digits. For example, the number 15 is made up of a 1 and a 5. The number 247 is made up of a 2, a 4 and a 7, etc. As we are dealing solely with the (10) digits 0 through 9 for displaying every number we work with, we are dealing with a number system that has a base (a foundation) of 10 unique numbers (or symbols), and therefore we are dealing with a decimal number system.

(.....more detail is in the book.....)

(66) Writing out hundreds, thousands, millions, billions, etc., for decimal numbers:

I want to revisit the writing out of large numbers, something we did in section # 15. You write out a large number in words, then have your child read the sentence, and write out the number. The only difference here is that the numbers will now contain a decimal.

The process for writing out large numbers that are in decimal format is very similar to what we have already learned.

Example 1: Write out "2.06084 quadrillion".

Here's the verbalization:	*Here's the answer build-up*:
Start with the whole number 2, and say "quadrillion".	2
Now, continue writing out the other numbers in groups of 3, from left to right, while verbalizing them also from left to right.	
- write out 060, and say "trillion";	2 060

(.....more detail is in the book.....)

(75) Summary - fractions, decimals, percentages:
As you have no doubt noted, fractions, decimals and percentages are all inter-related, and are just different ways of identifying the same number. For example, the number 7 can be shown as 7 (whole number) or as 7.00 (decimal format) or as 7/1 (fraction format) or as 700% (percentage format). The number 1/2 can be shown as 0.5 (decimal form) or as 1/2 (fraction form) or as 50% (percentage form).

The following tables summarize the conversions.

-- ***FRACTION***: --

	To percentage: •fraction x 100**%**	To decimal: •show the numerator with .00 •then perform the division	Summary:
1/2	1/2 x 100% = 50%	1.00/2 = 0.50	1/2 = 50% = 0.50
1/4	1/4 x 100% = 25%	1.00/4 = 0.25	1/4 = 25% = 0.25
3/4	3/4 x 100% = 75%	3.00/4 = 0.75	3/4 = 75% = 0.75

(.....more detail is in the book.....)

(76) Increase, increase-fraction, increase-percentage:
Let's look at increases in numbers, and how they relate to the original number. Some examples will make this topic clear.

Problem 1: Instead of receiving \$10, I received \$12.
Q: What is the increase, the increase-fraction and the increase-percentage I received?

Okay - so instead of \$10, I got \$12. That's an increase of \$2 over the original \$10. Now, the increase-fraction and the increase-percentage are both stated in comparison to the original number.

The increase-fraction is: the increase received, "divided by" the original number. Therefore, the increase-fraction is 2/10 which reduces to 1/5. I can now say that I received 1/5th more than expected (on the original amount).

The increase-percentage is the increase-fraction x 100%. Therefore, the increase-percentage is 2/10 x 100% = 20% (or, 1/5 x 100% = 20%). I can now say that I received 20% more than expected (on the original amount).

(.....more detail is in the book.....)

(77) Decrease, decrease-fraction, decrease-percentage:
Let's look at decreases in numbers, and how they relate to the original number. Some examples will make this topic clear. I will use the same problems as before, but with a "decrease" instead of an "increase".

(.....more detail is in the book.....)

(79) Rounding:
This is yet another topic that your child won't be able to escape in school. It could prove a tiny bit tricky, so read this closely.

"Rounding" a number means "approximating" the number. The "rounded" answer must remain very close to the original. As a quick example: We can all agree that 57 is closer to 60 than it is to 50. So, if I was asked to "round off 57 to the nearest 'tens' position", my answer would be 60. Note that 60 is the approximation. The original number is 57. The approximated number must end up close to the original, or something is wrong with your "rounding".

Before we look at the mechanics of rounding off any number to any "position", let us first take a quick review of digit "positions".

(.....more detail is in the book.....)

Problem 1: Round 123456.789 to the nearest "tens" position.
Step-1: Locate the digit in the position that you want the rounding done to. We want the rounding done to the "tens" position. We know that the digit "5" is in the "tens" position. So we want the rounding done up through and including the digit "5".

Step-2: Check out the digit after (to the right of) the rounding-digit "5". It happens to be the digit "6". I call this digit the "**Control**"-digit!
Please note that "Control" is my own made-up word for this digit. You may not find this word "Control" in other math books.

Step-3: Ask yourself: Is the Control-digit "5" or greater? The answer can only be Yes or No. Now memorize the following:
a. Regardless of whether or not the Control-digit is "5" or greater (that is, regardless of your answer being "Yes" or "No"), the Control-digit and all other digits following the Control-digit will drop down to zero.
b. If the answer is "no", then the rest of the number will remain the same!
c. If the answer is "yes", then the rest of the number will increase by the number "1" applied at the rounding-digit!

(.....more detail is in the book.....)

(80) Scientific notation:
So far, we have seen numbers written as whole numbers, decimals, fractions, or percentages. Numbers that are in the form of whole numbers (which have an implied decimal point after the number), or decimal numbers are all numbers written in "standard notation". For example: 12345; 1.2345; 0.0345; 287.99; etc., etc.

So even if you had to write very large numbers or very small numbers, you would normally write these in standard notation. For example, 6,000,000,000 (6 billion) is a very large number and 0.0000000006 (six ten-billionths) is a very small number, both written in "standard notation". There is another way of expressing very large or very small numbers.

Scientific Notation (I'll call it "SN", for short.....) is a format for expressing very large or very small numbers. *When used in conjunction with "Rounding", SN allows us to quickly appreciate the magnitude of very large or very small numbers*. I taught SN to Grandson as a challenging mental exercise.

The SN asks for any number to be written in the form: $n.nnnnn \times 10^{n}$

Note that this form requires: a number from **1** through 9; followed by a decimal-point; followed by one or more digits from **0** through 9; the resulting decimal number is now multiplied by a power of 10. Study the above format.....

(.....more detail is in the book.....)

(81) Permutations and combinations:
I taught this math topic to Grandson just as a mental exercise for cognitive reasoning. In a nutshell, permutations and combinations are both about "physical arrangements" of items or events. Here is the basic difference between arrangements that are permutations and arrangements that are combinations.

*P*ermutations (with a "*P*") are concerned with *P*ositions (with a "*P*"). Combinations are not concerned with positions.

As a quick example for now, you can permute (arrange with positioning) 3 items named A,B,C as follows: abc, acb, bac, bca, cab, cba, and these are all considered unique and distinct permutations. Because permutations are concerned with "position", it follows that abc, acb, bac, bca, etc. are all considered to be different permutations.

However, you can only combine A,B,C in just one way, regardless of whether you call it abc, acb, bac, bca, etc. This is because abc, acb, bac, bca, etc., are considered one and the same combination. Combinations are not concerned with "position". Think of combinations as mixtures. If you have a mixture of salt and sugar, it hardly matters to you how the salt and sugar particles are aligned with one another. They are still all in the same mixture. That's how it is with combinations.

(81-A) Permutations:

Problem 1: How many permutations (arrangements that are concerned with position) can you have of the letters A,B,C taken all at a time?

(.....more detail is in the book.....)

(82) The "BALANCE" concept in math - crucial!:

In attempting to teach Grandson how to quickly understand and solve "word problems", I coined the term "the balance concept in math". This concept, if properly utilized, makes short work of solving word problems, because most word problems fall under the "balance" concept, as explained in detail in this section. Have your child concentrate on this section. It will help make math life quite a bit easier. The balance concept can be applied in a variety of ways, and I will explain these as we move along.

(82-A) The basics of the "Balance" concept:

Problem 1: 4 books cost $12; how much would I pay for 16 books?
or: How much would I pay for 16 books if 4 books cost $12 ?
or: Calculate the price of 16 books at the rate of 4 books for $12, etc.

Regardless of how the problem is worded, there are clearly 2 sections in this problem:
(1) the data (or "balance") section, that says that "4 books cost $12"; that's given to us; there is no question posed there;
(2) the problem section, which asks how much I would pay for 16 books; this is the portion to be solved.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(82-B) "Balances" as fractions - very useful!:

So far, we have been showing "balances" as follows:

(a)	9 balances 72:	(b)	4 balances 20
so:	11 balances 88	so:	5 balances 25

regardless of what the "units" might be.

Now, let me show you another way of working with the balance concept in math. This method will greatly enhance your problem-solving ability. Study this section carefully.

Let's see the example of: 9 items cost $72
I can write this as: 9 balances 72 and I would be quite correct.

However, it is better to write balances as fractions! Note this explanation carefully!!

"9 items balance $72" is better written as:

$$\frac{9 \text{ items}}{\$72} \quad \text{or} \quad \frac{\$72}{9 \text{ items}}$$

9 items / $72 can be interpreted as: "9 items purchased, for every $72 spent" or "9 items for every $72"

$72 / 9 items can be interpreted as: "$72 spent, for every 9 items purchased" or "$72 for every 9 items".

(.....more detail is in the book.....this topic has been comprehensively covered.....)

Now, let's see how these new ways of writing "balances" can help us solve word problems faster.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(83) Word problems - every student's nemesis:

- but not yours - you know the "Balance" concept in math!

This section provides more practice in solving word problems.

1: How many bags of sugar can I purchase for $20 at the rate of 6 bags for $10 ?
Answer: $20 x 6 bags / $10 = 12 bags.
2: I need 12 credits for graduation. How many weeks of school are needed if I can get 3 credits every 12 weeks? Answer: 12 credits x 12 weeks / 3 credits = 48 wks.
3: I can make 2 cabinets every 5 hours. How many cabinets can I prepare if I work for 15 hours? Answer: 15 hours x 2 cabinets / 5 hours = 6 cabinets.
4: How many feet of tape did I use to wrap 5 presents, if I needed 2 feet of tape per present? Answer: 5 presents x 2 feet / 1 present = 10 feet.
5: I read 10 books every 5 months. I have read 30 books. How long have I been reading books? Answer: 30 books x 5 months / 10 books = 15 months.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(84) Unit and dimension analysis and conversion:

Believe it or not, but in all the "balance concept" problems you have worked out, you have been doing "unit and dimension analysis and conversion" all along, without the benefit of knowing the fancy title. In problem 1 of the previous section, you converted

$20 to 12 bags of sugar. So you converted "20" to "12" (that's dimension (or size) analysis and conversion) and you converted "$" to "bags of sugar" (that's unit analysis and conversion). This section merely extends on the balance concept by having you do <u>multiple conversions</u> at the same time.

We have already seen one example of this type earlier, when we converted 30 miles per hour to feet per second. The problems here are of a similar nature. You are given multiple balances, and you have to make multiple conversions based on the balances.

<u>*Problem 1*</u>: <u>Convert 50 miles to inches</u>, given that: 1760 yards make 1 mile; 3 feet make 1 yard; 12 inches make 1 foot.

Here I have been given a series of balances to analyze, and make conversions. I have to convert 50 miles to inches. Note that I have to convert the number "50" to some other number (that's "<u>dimension</u>" (size) analysis and conversion), and I have convert the unit "miles" to the final unit "inches" (that's "<u>unit</u>" analysis and conversion). Hence this topic is called "unit and dimension analysis and conversion".

(.....more detail is in the book.....this topic has been comprehensively covered.....)

<u>***(85) Areas and Volumes***</u>:

Every child needs to understand the basic concepts of "areas" and "volumes".

<u>***(85-A) Areas***</u>:

The word "Area" refers to the <u>size of a flat (2-dimensional) surface</u>. The word "area" asks <u>how large</u> is a flat surface (floor, wall, ceiling, table-top, etc.), or <u>how large is the space</u> enclosed by the boundary of a piece of land or a drawing such as a rectangle or square or a circle. The "flat" surface could be horizontal or vertical or oblique, that does not matter. For example, you could measure the flat surface area of your writing desk (which is horizontal) or your bedroom wall (which is vertical) or your ceiling (horizontal but upside-down) or the staircase in your home (which is oblique).

(.....more detail is in the book.....)

<u>***(85-C) Volumes***</u>:

Volume is defined as the <u>amount of 3-dimensional</u> (3-D) physical space that a 3-dimensional object occupies. Let's clarify this further.

1. There is a bathtub that is filled to the brim with water. You slowly enter this bathtub and you fully submerge yourself in the water. The water displaces, of course, and runs out of the tub. Ignoring the mess it creates, the quantity of water that runs out is equal to the <u>volume</u> of your body.

2. You have a cup containing some water, and you slowly introduce a large stone into the water until the stone is fully submerged. The water level in the cup rises. The volume of water displaced is equal to the <u>volume</u> of the stone.

What about the cup mentioned above? What is the volume of the cup? Note this very carefully. The volume of any container is generally understood to be the "volume that the container can <u>hold</u>", and <u>not</u> the physical amount of space that the container itself occupies. If the container is large (or small), it will <u>hold</u> more (or less) liquid, and therefore its "volume" would be considered accordingly to be larger (or smaller).

<u>The numerical answer for any volume must be followed by the words "cubic (units)"</u>. This is explained later, as we move along here. Look at figure 11.

(.....more detail is in the book.....)

<u>(86) Some interesting mental math</u>:
Let's try some interesting mental math.

<u>(86-A) Squares and products of larger numbers</u>:
1. What is the square of 99 ?
Note: The "difference" between 99 and the next "round up to 00" (which is <u>100</u>), is 1.

Proceed as follows: 99 - 1 = 98 This gives us the first-2 digits of the answer: <u>98</u>
Now square the difference (1), but show it as a *<u>2-digit number</u>* (because the original number being squared is a 2-digit number). This gives us the next-2 digits of the answer: <u>01</u>
Final answer: 9801

2. What is the square of 98 ?
Note: The "difference" between 98 and the next "round up to 00" (which is 100), is 2.
Proceed as follows: 98 - 2 = 96 This gives us the first-2 digits of the answer: <u>96</u>
Now square the difference (2), but show it as a 2-digit number (because the original number being squared is a 2-digit number). This gives us the next-2 digits: <u>04</u>
Final answer: 9604

(.....more detail is in the book.....)

Let's try squaring again, this time with higher numbers. It's the same logic as shown above.

1. What is the square of 999 ?
The "difference" between 999 and the next "round up to 000" (which is <u>1000</u>), is 1.
Proceed as follows:

999 - 1 = 998 This gives us the first-3 digits of the answer: 9 9 8
Now square the difference (1), but show it as a 3-digit number: 0 0 1
We show the square-of-the-difference as a 3-digit number because the original problem being squared is a 3-digit number. Final answer: 998001

(.....more detail is in the book.....)

(86-B) Fast multiplication with 11:

1. What is 15 x 11 ??

The first digit of the answer will be "1", which is the first digit in 15
The last digit of the answer will be "5", which is the last digit in 15
The middle digit will be the sum of the 1 and the 5, which is 6.
Final answer: 1 6 5

2. What is 27 x 11 ?? Answer: 2 9 7

3. What is 37 x 11 ?? *Temporary* answer: 3 10 7

But - I cannot write "10" as a *single* digit number in the 2nd column.
So, we write "0" for that column, and carry-over the "1" to the next (left) column.
So the first digit is now 3 + 1 = 4; the middle digit is now 0; the last digit remains 7.
Final answer: 4 0 7

(.....more detail is in the book.....)

(87) Basic Algebra:

Algebra is a branch of math that is useful in every branch of science and mathematics, and it is imperative that every child should have some background in basic Algebra.

The (Anglicized) word "Algebra" comes from a Latin variant of an Arabic phrase "Al Jabr", which translates to "the transposition". We will soon see (in the next section) what "the transposition" is all about, but for now I want to summarize the origin of the word "Algebra". So, the science (or the Math) of Algebra ("Al Jabr") is a contribution of the ancient Arabs.

Algebra works with numbers and symbols to express mathematical ideas. The numbers are called "constants", because their values are fixed. For example, 9 is 9, however you look at it. Its meaning and value are fixed, or constant. It's the same for any other number, positive or negative. Symbols, on the other hand, are unknown quantities that need to be resolved, and are considered "variables" or "unknowns" until resolved. Algebra solves for the symbols, thus bringing value and meaning to the symbols. Before we can embark on the study of basic Algebra, we have to learn some rules associated with expressing numbers and symbols, so that you can understand someone else's Algebra, and, more importantly, everyone can understand <u>your</u> Algebra. We will use numbers (constants) where appropriate, and we will use "x", "y", z" to denote <u>variables</u> where appropriate. You can really use any letter of the alphabet to represent "the unknown or the variable", but traditionally, "x", "y" and "z" have been favorite unknowns in mathematics, and mathematics honors tradition. So for the most part we will stay with x, y, and z to represent our unknown quantities in Algebra.

(.....more detail is in the book.....)

<u>(87-A) Rules for writing numbers and symbols in Algebra</u>:

Let's see some of the rules for writing numbers and symbols in Algebra:

1. <u>The Variable</u>: Any quantity whose value can change or whose value is unknown is called a "variable". Variables are usually assigned the letters x, y, z. However, any letter can be used. A variable by itself, such as "x", implies **<u>1</u>**x. We don't need to write the number 1 in front of the variable each time. If you do write 1x, then it implies that you are writing down: 1 * x ("1 multiplied by x").

2. <u>The SUM of 2 unknown quantities</u> is written as: $x + y$ or $y + x$.
What is the value of $x + y$?? I will not have the answer, unless you attach specific values to x and to y. Unless you attach some relevance (values) to x and y, I have no choice but to say that the sum of x and y remains as $x + y$ or $y + x$.

Eg: $4 + 5 = 9$; $5 + 4 = 9$; those problems had definite answers because we were working with known values 4 and 5. But if I now say: $x + y = 9$ or $y + x = 9$ then I can have an infinite number of combinations of values for x and y which add up to 9. So if you give me at least one of the values, either for x or for y, I can solve for the other unknown, but if I don't have any value for the x or the y, then the expression $x + y = 9$ will have to remain as $x + y = 9$.

3. <u>The SUM of "like" (similar) terms</u>: In Algebra, a+a is the same as 1a+1a, which makes 2a. Therefore, $a+a = 2a$. $y+y+y = 3y$. $x+x+x+x = 4x$ etc.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

14. <u>Resolving mathematical expressions</u>: Just so that everyone gets the same answer when tackling the same problem, there are rules to be followed for resolving math expressions. The rules indicate the <u>sequence of operations</u> that should be followed by everyone, to arrive at the same answer. *The rules are particularly useful when you have to deal with expressions that have no parentheses for clarification of what is needed*. But regardless of whether or not the expression has parentheses in it, you must always follow the rules, to resolve math expressions correctly.

The rules are remembered through the use of the acronym PEMDAS which tells you in what order you need to perform mathematical operations on an expression to get correct results. PEMDAS stands for:

Parentheses; **E**xponents; **M**ultiplication; **D**ivision; **A**ddition; **S**ubtraction.

PEMDAS: is to be interpreted as follows:

- resolve the **P**arentheses first (if any)
- resolve the **E**xponents next (if any)
- resolve the **M**ultiplication or **D**ivision next (if any), in the order of appearance in the problem, from left to right!
- resolve the **A**ddition or **S**ubtraction next (if any), in the order of appearance in the problem, from left to right!

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(87-C) Equations:

A common use of Algebra is to work equations to resolve an unknown quantity. We have seen many equations before in this book.

For example: no one will dispute that 5 + 4 = 9. Here we have known quantities, and there is a balance here, clearly depicted by the "equals" sign. The "equals" sign states that the left hand side (*LHS*) of the equation is equal to (or balances) the right hand side (*RHS*) of the equation. Therefore, 5 + 4 = 9 is an "equation". Other examples of equations could be: $a + b = c$ or $(7-x) / (a-b) = (z-x)$ etc.

Now, if you want to maintain the balance of an equation, then either you do nothing to the equation, or, if you must change the equation, then you must make the same change to both sides of the equation.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(88) Basic Geometry:

The word "Geometry" derives from the Greek words "geo" (meaning Earth) and "metria" (meaning "measurement"). The Greek mathematician "Euclid" is recognized as the Father of Geometry for his contribution to this branch of math. In a nutshell, Geometry is concerned with the size, shape and inter-relationships of 2-dimensional (2-D) and 3-dimensional (3-D) objects and figures. The study of 2-D objects and figures is called "Plane" Geometry, and the study of 3-D objects and figures is called "Solid" Geometry.

In this section on Geometry, we will concern ourselves with a small subset of "Plane" Geometry. There is also a branch of "Plane" Geometry called "Coordinate" Geometry, which studies relationships between geometrical shapes and Algebraic values and Algebraic expressions, and we will see some of that in the next section. The sole purpose of introducing these topics to Grandson was to get him to think logically and to reason.

(88-A) Basic definitions:
Because Geometry concerns itself with the shapes of objects and figures, let's start off with some very basic definitions and concepts that you need to impart to your child.

1. "2-dimensional": 2-D implies that we are looking only at the "length" and the "width" (or "breadth") of objects, and we are not at this time concerned with the "depth" (the 3rd dimension) of the object. For example: You are reading this text either on paper, or on a computer screen. You are seeing everything across the length of the paper or the screen, and across the height (or width) of the paper or the screen. There is nothing for you to see or appreciate in the "depth" of the paper or the screen, which you cannot access. You are, therefore, reading this in 2-D.

2. "Plane" Geometry: We will be studying geometrical patterns and shapes that will be drawn on flat surfaces (2-D surfaces) such as a sheet of paper, or a writing board at school, or a computer screen, or some electronic gizmo that your child may be using, etc. Now, "flat surfaces" are referred to as "Planes", and therefore our study of Geometry using figures drawn on flat surfaces is called Plane Geometry.

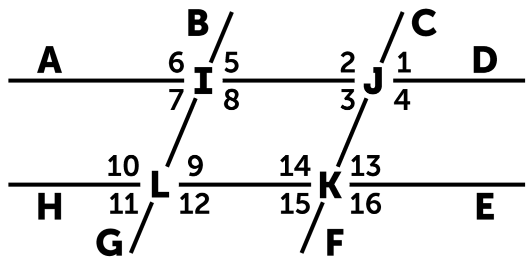
Emphasize the meaning of "plane" to your child, by running your hand over the screen surface, or over the page you are reading this on, or over a writing board, or over a book-cover, or over a table-top, etc., so that your child can appreciate the meaning of a "flat surface", or a "plane" surface. Also, ensure that your child does not get the impression that a "plane" surface is one that is either vertical or horizontal. For example, take a book in one hand and run your other hand over the cover of the book, while emphasizing the word "plane". Now, rotate the book at different angles, and again run your hand over the cover, while emphasizing the word "plane". Bottom line: Ensure that your child understands the concept that a "plane" can be any flat surface at any angle in relation to the horizontal or the vertical.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(88-E) Parallel lines crossing 2 or more other parallel lines:
Let's consider angle properties that become evident when we have multiple parallel lines crossing one another. Note that the lines considered are all "straight", and the

emphasis here is also on the word parallel. The lines must be parallel and straight, for the following angle properties to be true.

See figure 24.



(Fig. 24)

In figure 24, we have 2 parallel straight lines AD and HE, and they are cut by 2 other parallel straight lines BG and CF. Now, the following properties become apparent:

(A) Vertically opposite angles are equal: We have seen several examples of this. Any time 2 straight lines cross each other, vertically opposite angles become equal. In figure 24, the angle-pairs: $\angle 1$ and $\angle 3$; $\angle 2$ and $\angle 4$; $\angle 5$ and $\angle 7$; $\angle 6$ and $\angle 8$; $\angle 13$ and $\angle 15$; $\angle 14$ and $\angle 16$; etc., are vertically opposite angles equal to each other.

(B) Corresponding angles are equal: Look at $\angle 1$ and $\angle 5$. If you hold 2 pencils together at the point J such that they resemble $\angle 1$, you can now move your hand over to the point I, and the 2 pencils will resemble $\angle 5$. Simply put, imagine $\angle 1$ moving slowly across the page, and coming to rest on $\angle 5$. Can you visualize that the two angles resemble each other, except for their physical positions on this page? Yes, it is relatively easy to visualize $\angle 1$ slowly moving across the page and resting itself on $\angle 5$. Therefore, $\angle 1$ and $\angle 5$ are said to be "corresponding angles", that is, they "correspond" to each other, and are therefore equal. $\angle 1$ and $\angle 5$ are essentially the same angle, at different points on the page.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

(88-H) Theorem 1:

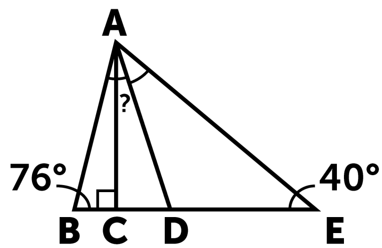
The sum of the 3 angles of any triangle always adds up to 180 degrees.

Regardless of the shape of the triangle, and the kind of angles it may have (all acute, or 1 obtuse angle, or 1 right angle), the sum of the 3 angles will always be 180 degrees. To prove this: Consider the triangle in figure 32, with angles A, B, C.

Construction: I am going to draw a small "construction" on the triangle that does not in any way, shape or form alter the triangle itself, but brings other facts to light. My "construction" is a simple line drawn through ∠A, that is parallel to the line BC. So our triangle now looks like this. See figure 32.

(.....more detail is in the book.....)

Q3: See figure 33.



(Fig. 33)

∠B = 76^0; ∠E = 40^0; ∠C = 90^0; ∠BAD = ∠DAE. Q: How large is ∠*CAD*?
Answer: ∠A + ∠B + ∠E = 180 degrees.

(.....more detail is in the book.....)

(88-I through 88-S: Other important theorems and concepts in Geometry):

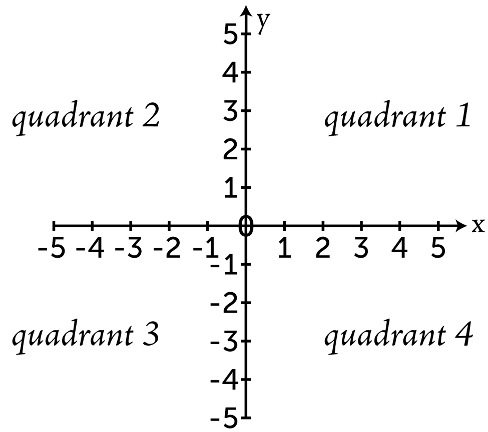
(.....more detail is in the book.....these topics have been comprehensively covered.....)

(89) Basic coordinate Geometry:

"Coordinate Geometry" is the math that studies relationships between Geometrical shapes and Algebraic values and expressions. It's a marriage of Algebra and Geometry.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

We are now going to take a big leap forward in conceptualization. We will add another number-axis to our horizontal number-axis, this one moving up and down in the vertical direction. See figure 52:



(Fig. 52)

Note the following points in connection with figure 52.

Up until now, we have only concerned ourselves with one number-axis, the <u>horizontal</u> number-axis, or what we will now call the <u>X-axis</u>. We are now introducing another axis in our conceptualization. This axis runs in the vertical direction, and we call this the <u>Y-axis</u>. The X-axis and the Y-axis meet at the Origin (point zero). Points on the Y-axis represent numbers, just as do points on the X-axis. Positive numbers on the Y-axis are <u>above</u> the origin, proceeding to plus-infinity; negative numbers on the Y-axis are <u>below</u> the origin, proceeding to minus-infinity. On the X-axis, you move to higher numbers when moving to the right, and to lower numbers when moving to the left. On the Y-axis, you move to higher numbers when moving <u>up</u>, and to lower numbers when moving <u>down</u>. The "scale" of the Y-axis (how far <u>spaced-apart</u> are the points) usually is the same as that for the X-axis (but does not have to be). In our diagrams, the X-axis and the Y-axis will have the same scale.

(.....more detail is in the book.....this topic has been comprehensively covered.....)

<u>(90) Basic important concepts in general science</u>:
Every young child should be aware of the following important scientific concepts.

<u>(90-A) Law of conservation of mass and energy</u>:
This law neatly sums up the fact that "you cannot get something for nothing".
The Law of Conservation of Mass and Energy provides a very important concept that everyone needs to understand and appreciate. I have had Grandson repeat this law to me several times. The actual statement of this law may vary slightly from one science text-book to another.

"The Law of Conservation of Mass and Energy states that the total amount of Mass and Energy in the universe is constant. Mass and Energy can be neither created nor destroyed. Mass and Energy can only be converted from one form into another."

In other words, there is no such thing as a "freebee", in the physical universe.

(.....more detail is in the book.....)

(90-B) Einstein's Equation:

$E = MC^2$ This is probably the most famous equation in the world.

This equation unites "mass" and "energy". The equation states:

(.....more detail is in the book.....)

(90-C) Difference between "mass" and "weight":

This is also a useful concept to impart to the young mind, and I run this by Grandson frequently.

The mass of an object is the "amount of matter" contained in the object. By "matter", we mean the atoms and molecules making up that object. If there are 10^{20} molecules in object-1, and 10^{30} molecules of the same type in object-2, then we can say with confidence that "object-2 is more massive than object-1", and object-2 will be physically larger than object-1. If objects 1 and 2 have the same number of, but differing types of, molecules, then we can say that one object will be more massive than the other, because its molecules are more massive than those of the other object.

(.....more detail is in the book.....)

(90-D) Difference between "heat" and "temperature":

We have seen in section # 90-A that there are many forms of "energy" - electrical, chemical, mechanical, heat, light, nuclear, etc. Heat, therefore, is one of the many forms of energy available in the universe.

Heat is a measure of the "energy level" of a body. This energy-level refers to the total "kinetic" (meaning "from movement") energy content of the vibrating atoms and molecules contained in the mass of the body.

(.....more detail is in the book.....)

(90-E) Electromagnetic induction:

There should be no doubt in anyone's mind that without the energy form known as "electricity", this world would be thrown back into the stone-age. We cannot live the way we are accustomed to, without the practical application of electrical energy. Homes (and just about everything in them), factories, offices, banks, computer installations, hospitals, schools, restaurants, planes, trains, automobiles and anything else you can think of all depend on electrical power to keep them going. If you have ever had a power outage affect your home, you know exactly how frustrating and upsetting that can be.

Our practical use of electricity would not have been possible without the contributions of an English scientist named Michael Faraday who discovered the phenomenon called "electromagnetic induction". I think that a small introduction to this topic would be very beneficial to every child.

"Electromagnetic induction": In a nutshell, it means that electricity and magnetism are two sides of the same coin. In other words, it takes magnetism to induce (create / produce) electricity, and it takes electricity to induce (create / produce) magnetism.

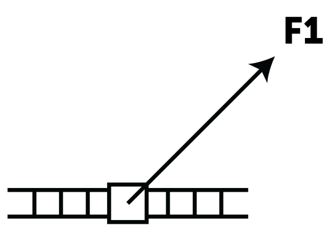
Michael Faraday's experiments showed that an electrical force is induced (created, produced) in a metal conductor if there is any relative movement between the metal conductor and a magnetic field. See figure 70.

(.....more detail is in the book.....)

(90-F) Composition and resolution of forces:

This topic deals with the summation of forces into a single force, and the separation of a single force into its components.

Look at figure 71. We have a railroad "boxcar" sitting on railroad tracks. We have a gang of men pulling on the box-car with a rope. In effect, there is a "primary force" acting on the box-car as shown in the diagram, acting in the direction that the rope is pulled. Let's say that the magnitude or strength of this primary force is F1 (regardless of the "units").



(Fig. 71)

Let's say the boxcar moves forward in the direction of the tracks.

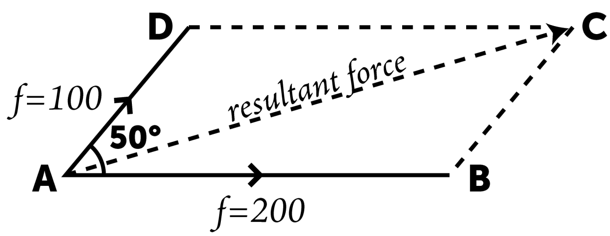
Now read this statement carefully. I say to you (and this is an indisputable fact of

Physics) that the boxcar will not move in the direction of the tracks unless there is a force acting on the boxcar in the direction of the tracks! ..... But there is no primary force acting on the boxcar in the direction of the tracks..... So why does (and how can) the boxcar move on the tracks in the direction of the tracks?

(.....more detail is in the book.....)

The parallelogram of forces:
To find the resultant of 2 forces acting on a body, we use a construction called "the parallelogram of forces". (There are also math formulas for this, but for now let's stay with the diagram itself, for a better visual understanding of this concept.)

See figure 78. Let's say that there are 2 forces acting on a body at point A. The forces have magnitudes of 200 units and 100 units, and they are acting at an angle of 50 degrees relative to each other.



(Fig. 78)

The single resultant force now acting on the body, that comes out of combining the 2 primary forces, can be obtained by drawing a parallelogram to scale such that it represents the problem at hand, and then measuring its diagonal.

(.....more detail is in the book.....)

(90-G) Composition and resolution of velocities:
The word velocity is very similar to speed, except for the main difference being that velocity refers to "speed in a specific direction", whereas speed is just a quantity. Velocity implies quantity and direction. So, you would say that "my speed was 50 miles-per-hour", but you would say "my velocity was 50 miles-per-hour north-northeast", etc.

In normal everyday usage, we don't use the word "velocity". We simply use the word "speed". I have yet to come across a cop who said to me: "Sir, your *v-e-l-o-c-i-t-y* was clocked at 80 miles-per-hour 15 degrees north of north-east"..... :):) But in the world of science, we have to say "velocity" when referring to speed in a specific direction.

(.....more detail is in the book.....)

Example 1: Nina is driving East at 60 miles-per-hour (mph). A 2nd car is traveling in a direction 50 degrees to the North of East, at a speed of 75 mph. When Nina turns to look at the 2nd car, what does Nina think is the "observed" or "apparent" velocity (speed and direction) of the 2nd car?

(.....more detail is in the book.....)

(90-H) What is color?:

I plugged in this small section as a last-minute thought. It was triggered by my reading about a competition in which entrants were asked to define the words "color" and "flame".

It would be difficult to define "color" in just one sentence, because the discussion of color spans the realm of physics, chemistry, and human anatomy and physiology. But if I have to attempt to put it in one sentence, I would say something to the effect that

(.....more detail is in the book.....)

Now, we can "see" things only if the following conditions are met:

1. We have eyes that are not damaged, and are functioning well.

2. Light from an object must reach the eye, in order that we may see the object. Note that we cannot see light itself. We can only see the effect of light on objects around us. So light must fall on an object, then bounce off the object and reach our eyes, for us to be able to see the object.

3. The light from the object must be scattered light, for us to see the object. It is impossible for us to see perfectly-transparent or perfectly-reflecting objects.

(.....more detail is in the book.....)

What is "Light"?

The concept of "light" has puzzled scientists for many years, and over the centuries many theories have been floated as to what constitutes light. Light is **something** that helps us see. Keeping this down to a very rudimentary discussion, let me say that there are basically two popular theories of light, one which states that light is a type of electromagnetic radiation that travels in the form of "waves", and the other which states that light is a type of electromagnetic radiation that travels in the form of "particles" called "photons". The one point of agreement is that "light is a type of electromagnetic radiation".

(.....more detail is in the book.....)

(90-I) What is a flame?:
I plugged in this small section as a last-minute thought. It was triggered by my reading about a competition in which entrants were asked to define the words "flame" and "color". Some of the definitions of "flame" were quite outlandish and verbose. So I decided to plug in my definition of the word "flame", something that I had learned in the 8th grade back in Bombay (Mumbai), India. I have run this by Grandson several times, too.

"A flame is a **gas** which is so hot that it gives out light."   (That's it - short and sweet.)

(.....more detail is in the book.....)

(90-J) Density:

(.....more detail is in the book.....)

The concept of "the Density of a substance" also has to do with the mass (weight) of the substance, but with one difference. Density is not just about the mass of an object. It's also about the volume of the object. (Recall that volume is in "cubic" units.)

Density is the mass per unit volume of a substance.
Mass per unit volume means the weight per cubic unit of the substance.

What this means is that if I can find out two things about an object (its Mass, and its Volume), I can divide the mass by the volume, and come up with the "density" of this object.

(.....more detail is in the book.....)

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

therefore, for any given mass of an object:
- the smaller the volume of the mass, the higher the density (the fraction value rises);
- the larger the volume of the mass, the lower the density (the fraction value falls).

Note the above statements. They tie in directly with the next section on "Anomalous expansion of Water".

(90-K) Anomalous expansion of Water:
This is a very interesting and important natural phenomenon that your child needs to be aware of. It ties in directly with the previous discussion on "Density".

Most substances expand when heated, and contract when cooled. If you place a metal object in a flame, the metal expands. The expansion may not be enough for you to appreciate it with the naked eye, but the expansion is there. Also, the force generated by the expansion is great. Railway tracks expand in the heat of the sun, and the tracks therefore have a "slack" feature at their joints that allows them to expand. Without the "slack" feature, the force of the expansion would be so great that the steel rails would bend out of shape, making rail travel impossible.

The bottom line is that most substances expand when heated and contract when cooled. Also, for most substances, the more they are heated, the more they expand and the more they are cooled, the more they contract.

This is true for most substances..... *but not for water*.....

(.....more detail is in the book.....)