P r e v i e w

Gifting Your Child:
a Foundation in Math

Kerman Bharucha

(This book is an extract out of my earlier book "*Gifting Your Child: Math and Literacy from Infancy*". If you have the earlier book, you do not need this one.)

This book provides a comprehensive introduction to basic math and science that every child needs to know. Starting with fundamental ideas, the book covers various detailed concepts in arithmetic, algebra, geometry, coordinate geometry and general science.

Garnering all this information from different books could prove to be problematic, but the information is all here, in this one book. All the topics are laid out in a logical progression in the table of contents, relieving the parent or guardian of the constant vexing question "What math do I teach my child next ?"

A solid foundation in math is paramount to a good education for all children because it will increase their overall academic confidence, help them maintain interest in math later in life, and prepare them for most careers which require at least some math proficiency.

This book will serve your child from Pre-K/K well into his 6th or 7th grade.

This preview gives you: (a) the full table of contents as it appears in the book so that you can appreciate the logical sequence in which the teaching of math has been approached and what topics have been covered; (b) extracts of the text from selected sections of the book so you can appreciate my method of explanation of the various concepts covered in the book.

I hope that this preview will convince you that this book will show you the way to giving your child or grandchild an invaluable educational head-start in life.

In the early stages of the child's life, when the child has not as yet learned to read, you have to be the teacher. This implies that you must be knowledgeable in at least some areas of math. ***This book will give you that basic math knowledge, which you can then impart to your child***. Once your child gets to be reasonably proficient at reading, he or she can continue on with the book with little or no assistance from you.

As this preview is an extract from the book, the section-numbers (preceding the section-names) will not be consecutive, and the page-numbers shown in the table-of-contents will have no corelation here, so there are no page numbers in this document.

Thank you for taking the time to preview my book.

Kerman Bharucha
Webster, NY (USA)

Gifting Your Child

a Foundation in Math

Kerman Bharucha

ISBN 978-0-9960474-5-6

Education is not the learning of facts. It is the training of the mind to think

\- *Albert Einstein* -

This book is geared towards several audiences:

(a) the parents of the pre-K child that needs a helping hand with math

(b) the young student in elementary or middle school, who has reading skills but has difficulty with basic math;

(c) anyone who wants a quick review of basic math.

The table-of-contents is a helpful guide to the specific topics discussed in the book.

Gifting Your Child
a Foundation in Math

<u>Table of Contents</u>

Appendices

Introduction:

I have always had a love of learning, although I have to admit that I was not a very good student. I first graduated from college in 1969 with a Bachelor of Science degree from the University of Bombay (Mumbai) India, then went back to college 30 years later for a 4-year degree in Physician Assistant from Rochester Institute of Technology, Rochester NY. I have also always had a love of teaching. I used to teach computer science for many years to employees of Xerox Corporation, I used to run my own software training classes, and I have also taught math and science to individual students over the years.

This book provides a very comprehensive introduction to basic math, including discussions of some key principles in basic science, that every child needs to know. Starting with the most fundamental of ideas, the book covers a wide progressive array of concepts in the areas of arithmetic, algebra, geometry, coordinate geometry and general science. Obtaining this information from several sources could prove to be problematic. But it's all available here, in this one volume. This book relieves the parent or guardian from answering the constant vexing question "What math do I teach my child next ?" If you follow the logical progression of steps as suggested in the table-of-contents, your child will be well on the way to a strong foundation in math. You can, of course, always pick specific topics to explore from the table-of-contents.

For the very young child who has not yet learned to read independently, you will have to be the teacher. This book will provide you with the knowledge you may need to teach math to your child. The young child who is able to read can continue on with the book with a little periodic assistance from you.

The bottom line here is that your child cannot escape his/her destiny with math, and a good background in math is paramount to his/her education. It is most important to give your child an early grounding in math because: (a) confidence in math will translate to overall academic confidence; (b) it is easier for the child to *maintain* a math interest in later life than it is for the child to *spark* a math interest in later life; (c) what the child learns in the first 5-10 years of his/her life will stay in place for the rest of his/her life; (d) the child will need to have some math proficiency in most career choices in his/her life.

As of now, I will refer to your child as "he" or "him", just to avoid the unnecessary "s/he" or "him/her" or "his/her" at every reference, which will only serve to waste time. (No, I am not a male chauvinist pig.....).

(1) Starting out early:
I am starting off this book on the premise that your young child is either at a pre-K level or is in Kindergarten or is in Grade-1 (or slightly beyond), and that *he is able to recognize, verbalize and write the basic numbers 0 through 9*. If this is not the case, I would suggest your taking a hiatus on this book, and resuming it when he has achieved the above milestone.

Of course, it is possible that your child is slightly older and is in a higher grade in school, and you don't need to start the child off at the very beginning of math concepts. In this case, please look through the table-of-contents and work on the topics you want to discuss with him.

For the purposes of this book, I will presume that your child is around the pre-K, or Kindergarten or Grade-1 level, and can comfortably verbalize and write out the basic numbers 0 through 9. We can now go ahead with learning higher numbers.

I want to emphasize a very important point, which is that *repetition and reinforcement are necessary tools for imparting certain concepts to a very young mind*. I will refer to this strategy often, throughout the book.

(2) Progression to numbers 10-99:
To progress the child to numbers 10-99, you need to *write out and verbalize the "duality" of the numbers*, then have your child do the same. Proceed as follows. Write out some numbers like 11 and 12 and 13, and say "one-and-one is eleven"; "one-and-two is twelve", "one-and-three is thirteen", etc., emphasizing to your child how two individual numbers generate a very different-sounding number when used together. Then have your child do the same for 11, 12, 13, by again verbalizing the "duality" of the numbers.

Next, emphasize the *symmetry* in verbalizing many numbers. For example, while writing out numbers like 25, 26, 27, say to him "2 and 5 make twenty-five, 2 and 6 make twenty-six, 2 and 7 make twenty-seven", emphasizing the common words "two" and "twenty" in the numbers, and have him repeat these back to you. Then make up a sequence such as 54, 64, 74, and say to him "5 and 4 make fifty-four, 6 and 4 make sixty-four, 7 and 4 make seventy-four", emphasizing the common word "four" in the numbers, and have him repeat these back to you.

If your child can appreciate that 7 and 4 make "seventy-four", then with some practice it would be very intuitive for him to say that 8 and 4 would make "eighty-four" or 7 and 5 would make "seventy-five". Take advantage of the symmetry found in the verbalization of numbers, and within a short time your child will be able to recognize and write out all 2-digit numbers.

Periodically, you will want to test his recall of 2-digit numbers, which can be easily done by showing him random 2-digit numbers and asking him to verbalize them, or by your verbalizing the numbers and asking him to write them down.

You should not proceed with further math concepts until your child can readily verbalize and write out 2-digit numbers.

(3), (4) and (5):
(I had to leave these sections blank, because the rest of this book is an extract from my previous book "*Gifting Your Child: Math and Literacy from Infancy*". Leaving the section numbers intact has facilitated a clean extract, for the purposes of this book.)

(6) Skip-counting:
Our next logical step with numbers will be to have your child count forward by 2s, then by 5s, then by 10s. You should skip-count quickly, then have your child repeat it. For example, say "2-4-6-8-10", or say "10-12-14-16-18-20", then have your child repeat that sequence. Do the same with skip-counting by 5s, and by 10s. With a bit of practice, your child will get through skip-counting quickly and correctly for the numbers 1-100. Restrict the skip-counting to counting by 2s and 5s and 10s, and in the forward direction only.

(7) Reverse counting:
Our next logical milestone is to have your child count backwards sequentially from any number up to 100. Start off with counting backwards from 10, then from 20, then from 30, etc. For example, say "10-9-8-7-6-5-4-3-2-1-0", then have your child repeat that. Or say "30-29-28-27-26-25", then have your child repeat that. With a bit of practice and given some time, he will get through reverse-counting correctly for the numbers 1-100.

(8) The limiting factor of "60" in clocks:
The "digital" age has negated the need for children to be able to read analog watches and clocks (those with hour/minute/second hands). However, it is my personal belief that the ability to read analog time-pieces is important because it encourages other abilities in the child such as logical thinking and memory development.

So at this time let us go ahead and explain to your child a crucial concept, that of "60" as the limiting factor in watches and clocks. But before we can do that, we have to introduce your child to time-pieces in general.

(.....more details in the original.....)

(10) Counting a clock down in reverse:
The next logical and crucial progression is to have your child count-down a clock in reverse, which of course is not the same as counting down numbers in reverse because of the limiting factor of 60. A good visual way to teach clock count-down to a child is to use the digital clock of a microwave oven. When heating food in the microwave (or by simply activating the "timer"), have your child verbalize the numbers as they count down. Caution your child to be alert as the clock approaches "zero" seconds. Within a short time, your child will be able to correctly state that a clock-time of 3:00 will become 2:59 in count-down mode.

(11) Understanding a directional compass: (long, but very necessary)
You may skip this section if you wish, but I have included this section in the book because I think it is very important that every child should have some basic knowledge of "directions" as an important concept to understand, not just for the sake of knowing directions in general, but because the effort made here will also help him greatly with "spatial visualization" and memory enhancement.
Here is how you may want to consider teaching your child about directions in general. Draw figure 1 as shown.

(.....more details in the original.....)

(12) Numbers 100 to 999:
Our next logical step should be to have your child understand numbers between 100 and 999. Because the decimal system is so symmetrical and repetitious, your child should not have major difficulty in grasping numbers greater than 100. Write down the number 101, and have your child repeat after you "one hundred and one". Now write down 201, and ask your child: "If 1-0-1 is one hundred and one, what do you think 2-0-1 would be?" Hopefully, you will get the correct answer. If not, backtrack a bit and try again. Then continue with 301, 401, etc. and have your child verbalize these numbers. Continue with other logical sequences of numbers such as 109, 209, 309, etc., or 199, 299, 399, etc., where the "hundreds" is emphasized, or 555, 556, 557, etc. where the "units" is being emphasized. If your child understands 5-0-1 to be "five hundred and one", then it should be most natural for him to refer to 6-0-1 as "six hundred and one" or to refer to 5-0-2 as "five hundred and two".

(.....more details in the original.....)

Stay on this path for as long as it takes your child to verbalize any 3-digit number without hesitation, because understanding all higher number concepts will depend on the child being able to verbalize 3-digit numbers rapidly.

(13) Units, tens, hundreds, and the place value of a digit:
These are very important concepts because they are a cornerstone to a clear understanding of much larger numbers, so take as much time as is needed for your child to understand the following concepts. By the time you are done with this section and the next one, your child will be able to verbalize numbers up to 36 digits long!

It is important that your child is able to readily verbalize 3-digit numbers. If this is not the case, take a temporary hiatus on this section, and review the previous sections. Write down a 3-digit number (say 123). At this time, your child should be able to readily verbalize this number as "one hundred and twenty-three". Point to the "3" and tell your child something to the effect: "This number '3' is in the 'units' position. 'Units' also means 'singles' or 'ones'. It means that there are three 'ones' (or 3-singles or 3-units) shown in this position. If you have 3 singles of something, you would have 3 of that something. Therefore, this digit '3' brings a value of '3' to the number 123." Emphasize the words "units / ones / singles" several times, and have your child repeat them.

Now point to the "2" and tell your child something to the effect: "This number '2' is in the 'tens' position. It means that there are two 'tens' shown in this position. If you have 2 tens of something, you would have 20 of that something. Therefore, this digit '2' brings a value of '20' to the number 123." Emphasize the word "tens" several times.

Now point to the "1" and tell your child something to the effect: "This number '1' is in the 'hundreds' position. It means that there is one 'hundred' shown in this position. If you have 1 hundred of something, you would have 100 of that something. Therefore, this digit '1' brings a value of '100' to the number 123." Emphasize the word "hundred" several times, and have your child repeat it.

Now comes the logical conclusion. The number 123 is verbalized as "one hundred and twenty three" because it is a combination of the values "one hundred" (provided by the digit 1) + "twenty" (provided by the digit 2) + "three" (provided by the digit 3). Hence the overall number is "one hundred and twenty and three", or simply "one hundred twenty three".

(.....more details in the original.....)

(14) Verbalizing hundreds, thousands, millions, billions, trillions.....:
Start on this section only if your child can effortlessly verbalize numbers up to 3-digits, and can identify the actual value of any digit in the number, depending on its position (place). If this milestone is achieved, we can now go ahead and take a giant leap forward in your child's understanding of numbers.

Write out any string of numbers as for example: 123456789234567890

Now start grouping them in 3s, from the right hand side. Your number should look something like this: 123 456 789 234 567 890

Tell your child that all numbers should be grouped in 3s from the right hand side, and it does not matter if the leftmost group has less than 3 digits.

Now we name the groups, starting from the right hand side. Taking as our example the same number above, the group-names (read them from right to left) are:

<<<<<-----<<<<<-----<<<<<-----<<<<<-----<<<<<-----<<<<<-----<<<<<--

123	456	789	234	567	890
quadrillion	trillion	billion	million	thousand	hundred

Therefore, the above number would be verbalized as "One hundred and twenty-three quadrillion, four hundred and fifty-six trillion, seven hundred and eighty-nine billion, two hundred and thirty four million, five hundred and sixty-seven thousand and eight hundred and ninety".

Note very carefully!   This point is crucial to understand!

In any group of 3 digits above, the rightmost digit is still "units", the middle one is still "tens" and the leftmost is still "hundreds". For example, in the above 567 (which is in the "thousand" group), the 567 by itself is verbalized as "five hundred and sixty-seven", but we add the qualifier "thousand" after that number, because the 567 is in the "thousand" group. It is not just 567 as a stand-alone 3-digit number, in which case it would only be "five hundred and sixty-seven" in value. This "567" brings "567 thousand" to the overall number.

In the above 789 (billion group), the 789 by itself is verbalized as "seven hundred and eighty-nine", but we add the qualifier "billion" after that number, because the 789 is in the "billion" group. It brings "789 billion" to the overall number.

As with any 3-digit number, the location of a digit in a number-group determines the actual value that digit will bring to the number-group. For example: in the "789 billion" group, the 7 brings a value of 700 billion to the overall number, the 8 brings a value of 80 billion to the overall number and the 9 brings a value of 9 billion to the overall number. Make sure your child understands this.

(.....more details in the original.....)

With some memorization, your child should be able to recite the group names rapidly from right to left (lowest to highest), <u>and</u> from left to right (highest to lowest). Have your child memorize 12 group-names as follows, forwards and backwards. They are written down in the text from highest (leftmost) to lowest (rightmost).

Decillion, Nonillion, Octillion, Septillion, Sextillion, Quintillion, Quadrillion, Trillion, Billion, Million, Thousand, Hundred.

Note that there are 12 groups, each with 3-digits in them, therefore with some practice and some memorization your child will be able to verbalize any number up to 36 digits long. Imagine the mental gyrations your child will be forced to go through to master this concept and memorize the 12 group-names from left to right and from right to left.

To summarize, for now:
1. always group digits in groups of 3 from right to left; the left-most group may have less than 3 digits;

2. now verbalize the group-names from right to left, and remember the group-name you end up on;

3. now verbalize the full number from left to right, starting with the leftmost group-name that you ended up on. This implies, of course, that the group-names have been memorized forwards and backwards.

Let's take some examples of numbers that your child can practice verbalizing. Remind your child to group numbers in groups of 3 digits from right to left before attempting to verbalize the number from left to right. Obviously, your child will have to memorize the group names forwards and backwards before he can identify the groups of 3-digits each, and then verbalize the overall number.

The following examples may read a bit awkwardly, but work on it. Name the groups from right to left, as indicated. Verbalize the overall number from left to right.

<u>Examples</u>:
1,234 <---- read R to L ----
The groups in 3s from right to left are "thousand / hundred", so this number is "one thousand / two hundred and thirty four".

12,345 <---- read R to L ----

The groups in 3s from right to left are "thousand / hundred", so this number is "twelve thousand / three hundred and forty-five".

1,000,234 <--------- read R to L ----------

The groups in 3s from right to left are "million / thousand / hundred", so this number is "one million / <u>no thousand</u> / two hundred and thirty-four".

- in reality, you don't need to verbalize "no thousand", but it will help your child to remember that there is a "thousands" group in there that should not be skipped over. If there is no value to any specific group, that group will be shown with 3 zeros. For now, I would suggest that your child should verbalize "no thousand", just for practice.

7,003,200,009 <-------------- read R to L ---------------

The groups in 3s from right to left are "billion / million / thousand / hundred", so this number is "seven billion / three million / two hundred thousand / and nine".

7,300,002,009 <-------------- read R to L ---------------

The groups in 3s from right to left are "billion / million / thousand / hundred", so this number is "seven billion / three hundred million / two thousand / and nine".

7,000,000,009 <-------------- read R to L ---------------

The groups in 3s from right to left are "billion / million / thousand / hundred", so this number is "seven billion / <u>no million</u> / <u>no thousand</u> / and nine".
(in effect, "seven billion and nine".....).

- again, you don't have to say "no million, no thousand", etc., but for now it will help your child to practice recalling the group names and to avoid skipping over any group.

(.....more details in the original.....)

And now for the big 36-digit enchilada.....

205,000,406,202,489,899,010,000,003,007,292,004

The groups in 3s from right to left are: (read both lines from R to L)
"decillion / nonillion / octillion / septillion / sextillion / quintillion / quadrillion / trillion / billion / million / thousand / hundred".

In the above number, the groups in 3s from right to left are all 12 group names given earlier, so this number is:

"two hundred and five decillion / no nonillion / four hundred and six octillion / two hundred and two septillion / four hundred and eighty-nine sextillion / eight hundred and ninety-nine quintillion / ten quadrillion / no trillion / three billion / seven million / two hundred and ninety-two thousand / and four".

(.....more details in the original.....)

(15) Writing out hundreds, thousands, millions, billions, trillions**.....**

At this point, let's take the following logical enhancement, and have your child write out large numbers. You spell out a large number in words, then have your child read the sentence, and write out the number. For example, you write out "Fourteen thousand and seven"; your child will read the sentence out loud, then write down "14,007" as the answer. Obviously, the child must know the group-names backwards and forwards, and the child must be able to read reasonably well before you can work on this section.

Presuming for now that your child knows the number-groups well and can read reasonably well, let's practice writing out large numbers.

Suppose the child has to write out "Eight trillion, seven thousand and four" as a number.

Step-1: Write down the number 8, and say it aloud as "8 trillion". So at this point, the number is simply: 8 (but in the child's mind it is 8 "trillion".)

Step-2: After the 8 "trillion", the next group should be "billion". But there is no "billion" mentioned in the number to be written out. So say "no billion" and put down 3-zeros, to represent the "billions" group. So at this point, the number is: 8 000

Step-3: After the 000 "billion", the next group should be "million". But there is no "million" mentioned in the number to be written out. So say "no million" and put down 3-zeros, to represent the "millions" group; at this point, the number is: 8 000 000

Step-4: After the 000 "million", the next group should be "thousand". We do have a thousands value in the number to be written out. It is 7. So say "seven thousand" and put it down as a 3-digit number 007. So at this point, the number is: 8 000 000 007 (Note the 7 shown as a 3-digit number.)

Step-5: After the 007 "thousand", the next group should be the "hundreds" group. We do not have a "hundred" specifically mentioned in the number to be written out, but we do have the number "four" mentioned. So we write it out as a 3-digit number 004.

The final number will now be written out as: 8 000 000 007 <u>004</u>

(.....more details in the original.....)

Try another example. Have the child read and then write out: "Seventeen septillion, five million and one"

Step-1:   write down the number 17, and say it aloud as "17 Septillion";
Step-2:   no "sextillion" mentioned; so say "no sextillion", and write down 000;
Step-3:   no "quintillion" mentioned; so say "no quintillion", and write 000;
Step-4:   no "quadrillion" mentioned; so say "no quadrillion", and write 000;
Step-5:   no "trillion" mentioned; so say "no trillion", and write down 000;
Step-6:   no "billion" mentioned; so say "no billion", and write down 000;
Step-7:   we <u>do</u> have "5 million" mentioned so say "5 million" and write down **00**5
Step-8:   no "thousand" mentioned; so say "no thousand", and write down 000;
Step-9:   no "hundred" mentioned, but the "one" is part of the lowest ("hundred") group, and will be written as ***<u>00</u>***1.

So the answer is: 17,000,000,000,000,000,005,000,001

(.....more details in the original.....)

Why do we need to work with children on such large numbers? After all, there may not be enough grains of sand on Earth to come close to a decillion. However, I think that it is very important to challenge every child's capacity to conceptualize, memorize, rationalize, spatialize, verbalize and any other *-ize* you can think of. Besides, consider the benefits of the mental challenges you would put your child through.

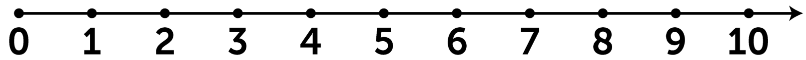
<u>(16) Defining "point", "line", "straight line"</u>:

We will take a small but crucial tangent here to explain the following very important definitions and concepts to your child. He needs to understand these concepts and mull them around in his head, and memorize these definitions.

1. <u>a point is that which exists, but has no dimension</u>. In other words, a point is an imaginary concept. However small a point you can think of in your imagination, I can break that up into a million bits, and each one of them would be a point. And each of those points can be further broken up again into a million pieces, each of which again would be a point, etc. Hence we have the concept of something that exists, but having no dimension. Think of the amount of imagination the child has to bring to bear on this concept alone. There is nothing like a bit of math to keep the mental gears grinding.

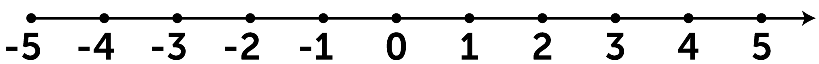
(.....more details in the original.....)

(17) The all-important "number axis" and the true meaning of Numbers:
You are now at a very important cross-roads in your child's education. You have to impart to him the true meaning of numbers. Your child needs to be very comfortable with this concept, to be able to work on the many other nuances concerning numbers. Let's start by drawing figure 4 as shown.



(Fig. 4)
(.....more details in the original.....)

(18) The concept of "positive" and "negative" numbers:
The concept of positive and negative numbers is yet another very crucial concept for your child to comprehend. You cannot do without this concept. Proceed as follows. Draw figure 5 as shown.



(Fig. 5)
(.....more details in the original.....)

(21) Addition:
"Addition" (+) simply means moving to the right on the number-axis. ("Subtraction" (-) (covered later) simply means moving to the left on the number-axis.)
(.....more details in the original.....)

(22) Addition involving negative numbers:
Look at the following problem:
9 + (-3) = ? This is the same as: 9 + -3 = ?
("What is the sum of 9 and -3 ?")
(.....more details in the original.....)

(23) Addition for the real world:
As useful as the number-axis is for a clear understanding of the many concepts of numbers, it is not a practical tool for daily usage. In school, for example, your child will be asked to add numbers such as:

```
  15      23      85      409      125      600      909      1010
 + 5     +35     +29     +  9     +103     +202     + 54     + 987
 -----   -----   ----    -----    ------   ------   --------  ---------
```

Now, taking one of the examples of 600+202, no one wants to start at point 600 on the number-axis and move to the right by 202 points to get to the answer. So while the number-axis is an excellent tool for getting the basic concepts understood, we need a more practical method for performing the above additions.

Before we start on the process of addition, emphasize the following to your child:

(1) You can go ahead with addition without consideration of the outcome. This statement will be made clear in the topic of "Subtraction for the real world", where you do need to take a quick "second look" at the problem, before starting on any subtraction process.

(2) Notice that all the numbers are right-justified, so the numbers line up on their right-most digit. In other words, the "units" digit of one number lines up under the "units" digit of the other number, and the "tens" digit of one number lines up under the "tens" digit of the other number, etc. In the above examples, note how the 15 and 5 are lined up, and how the 409 and 9 are lined up, and how the 909 and 54 are lined up, etc. The numbers are in columns, and the columns are justified (aligned) on the right.

(3) We always start the addition process from the right, and work our way to the left. And this is where the ability to add two numbers quickly with ready-recall is appreciated.

(4) We can only write 1 digit under any one column. This is clarified via examples.

(.....more details in the original.....)

(25) Subtraction:

"Subtraction" (-) simply means moving to the left on the number-axis. ("Addition" (+) (covered earlier) simply means moving to the right on the number-axis.)

(.....more details in the original.....)

(26) Subtraction involving negative numbers:

We have seen this before, but is worth repeating here.

(a) what is -7 - 4 ? Start at point -7 on the number-axis, then move to the left by 4 points, and we will come to point -11. Therefore -7 - 4 = -11.

(b) what is -15 - 5 ? Start at point -15 on the number-axis, then move to the left by 5 points, and we will come to point -20. Therefore -15 - 5 = -20.

(.....more details in the original.....)

(27) Subtraction for the real world:

As useful as the number-axis is for a clear understanding of the many concepts of numbers, it is not a practical tool for daily usage. In school, for example, your child will be asked to subtract numbers such as:

```
  15       23      85      409       125       600       909       1010
 - 5      -35     -29     -  9      -103      -202      - 54      - 987
-------  -------- --------  -------- ---------  -------   --------  --------
```

Now, taking one of the examples of 600 - 202, no one wants to start at point 600 on the number-axis and move to the left by 202 points to get to the answer. So while the number-axis is an excellent tool for getting the basic concepts of subtraction in place, we need a more practical method for performing the above subtractions.

Before we start on the process of subtraction, emphasize the following to your child:

(1) Never proceed blindly with a subtraction process. First look at the problem carefully, and ask yourself if the result will be a negative number, before proceeding with the subtraction. For example:

(.....more details in the original.....)

(28) Subtraction leading to a negative answer:

Remind your child: Before attempting a subtraction, take a quick second look at the numbers given to you. If you are attempting to subtract a larger number from a smaller one, the answer will always be negative.

(.....more details in the original.....)

(29) Direct subtraction leading to a negative answer:

This concept provides very good mental exercise for your child. He will have to think outside the box on this one. Look at the following subtraction.

(Problem 1:)

```
   3 8 4
-  9 4 3
---------
```

Because this is a subtraction, we will take a quick second look at the numbers given here. From "three hundred something" I am trying to subtract "nine hundred something". Clearly, the answer here will be a negative number. So we could proceed on this subtraction with a twist. So far, our "twist" has been to physically rewrite the larger number above the smaller number, and then do the subtraction in the normal way, but show the answer as a negative value in the original problem.

(.....more details in the original.....)

****************************   Tangent   *****************************

For the subtraction process, I have followed the method that if you take "help" in a specific column, you must add "1" to the number that is diagonally-across from the "helped" number in the next column over to the left. This method has allowed us to do normal subtractions and "reverse" subtractions (for negative answers) very easily, moving smoothly from column to column.

However, it is possible that your child may be taught a slightly different method in his school.

(.....more details in the original.....)

**************************   End of Tangent   **************************

(33) Multiplication:

As with the other concepts in math, this one is also crucial.

Explain to your child that "multiplication is merely a short-hand for repeated addition".

Say you want to add 7 to itself 10 times. You could express this as: 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7. And when you're done adding this up, you will hit the value 70. A much shorter and faster way to express "7 added to itself 10 times" would be: **7 x 10**. This is much shorter and faster than writing 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7.

(.....more details in the original.....)

(34) Multiplication for the real world:

In school, your child will have to deal with the following types of multiplication problems:

```
  4           27           2105           30297           13000
x 7         x  35         x   86         x 319             x    3
--------    ----------     ---------     ----------      -----------
```

Before we start on the process of multiplication, emphasize the following to your child:

(.....more details in the original.....)

(38) Multiples and factors:
If your child has the basics of multiplication down, now is the time to introduce two terms to him: multiples and factors.

The multiples of a number-n are the (larger) numbers you get by multiplying this number-n with other numbers.

(.....more details in the original.....)

(39) The concept of a "matrix":
Now that your child has a basic understanding of multiplication, the next logical concept for your child to grasp is that of the "matrix". While the word "matrix" has many meanings in science and medicine, in basic math terms a matrix is a symmetrical arrangement of items in the form of a square or a rectangle. The items could be anything: cans, bottles, fruits, flower-pots, coins, people, boxes, etc. Our objective here is to get your child to understand some simple math associated with matrices (plural of "matrix").

(.....more details in the original.....)

(40) Squares, cubes, etc., and roots:
The topic of "squares" and "cubes" builds upon the concept of "multiplication", but refers to a number multiplying itself a few times. This topic also reminds you that the language of math is a short-hand notation for expressing simple and complex ideas succinctly.

(.....more details in the original.....)

(46) Distributing items based on a ratio:
If you feel that your child is reasonably comfortable with basic multiplication and division, we should now look at the concept of "distributing items based on a ratio". This is a very interesting and useful concept that your child needs to know. Drum this concept into him.

Problem 1: You want to distribute 20 marbles between two kids, "in the ratio 2 to 3". How many marbles would each kid get ?

First, let us understand the meaning of the phrase "in the ratio 2 to 3".
A "ratio 2 to 3" means that if one person gets 2 of something, another person gets 3 of the same. Extending the concept, for every 2 of something given to person-A, you give 3 of the same thing to person-B. You continue this process until you are out of whatever it is that you are distributing.

(.....more details in the original.....)

(48) Fractions - proper:

"Fractions" is one of those critical topics in which your child needs to have a very thorough grounding. You would be surprised to know how many adults out there have difficulty even with the most basic concepts involving fractions, never mind the various nuances associated with fractions.

Check out the following numbers: $\frac{2}{3}$ $\frac{5}{7}$ $\frac{4}{9}$ $\frac{7}{11}$ $\frac{9}{17}$ $\frac{6}{13}$

These numbers are called "Fractions". I will explain very soon why they are called "fractions". For now, let's see the several ways of verbalizing these numbers.

Example 1: 2/3 can be verbalized as: "two upon three"; "two divided by three"; "two divided into three"; "two-thirds". But the best way by far to verbalize this number, so as to bring out its true meaning, is "two out of three".

Example 2: 5/7 can be verbalized as: "five upon seven"; "five divided by seven"; "five divided into seven"; "five-sevenths". But the best way by far to verbalize this number, so as to bring out its true meaning, is "five out of seven".

Example 3: 4/9 can be verbalized as: "four upon nine"; "four divided by nine"; "four divided into nine"; "four-ninths". But the best way by far to verbalize this number, so as to bring out its true meaning, is "four out of nine"; etc., etc.

(.....more details in the original.....)

Continuing on with our discussion of "Fractions":

Let us check out 2/3 again. I can refer to this as "two upon three", or as "two divided by three", or as "two divided into three", or as "two-thirds", or (the best way) as "two out of three".

Now let's understand this clearly. If I have 3 items in hand, and I gave you 2 out of the 3 items, I have not given you all 3 available items. I have only given you 2 out of the 3 items. I have only given you a portion or a fragment of the total number of available items. This portion or fragment is called a fraction. And because I can physically give you 2 items out of 3 items, "2/3" is called a "proper fraction" (also called "real fraction", or "true fraction").

(.....more details in the original.....)

(63) Decimals:
I know that you have heard this *ad nauseam*, but this topic (like every other topic in this book) is also very important for your child to understand. He cannot escape his destiny with the decimal system.

The word "decimal" derives from the Greek word "deca", meaning 10. Our familiar number system is called the "decimal" number system because it is founded on a base (a foundation) of only 10 unique numbers (or symbols). Note the word "unique".

The unique numbers (or symbols) that form the base of our number-system are: "0,1,2,3,4,5,6,7,8,9". The numbers 0 through 9 (that makes 10) are the only 10 unique numbers (symbols) that make up our number system. Any value beyond 9 has to be shown as two or more of the 10 base unique digits. For example, the number 15 is made up of a 1 and a 5. The number 247 is made up of a 2, a 4 and a 7, etc. As we are dealing solely with the (10) digits 0 through 9 for displaying every number we work with, we are dealing with a number system that has a base (a foundation) of 10 unique numbers (or symbols), and therefore we are dealing with a decimal number system.

(.....more details in the original.....)

(63-A) Other number systems:
Let me take a tangent here to introduce you to the interesting world of "number systems".

While we deal mainly with the decimal number system, in reality you can have a number system to any base you want. In the world of computer science, you will come across a "hexadecimal" number system (to base 16) and a "binary" number system (to base 2). But in reality you can have number systems to base 5 or 6 or 7 or any other base.

(.....more details in the original.....)

(63-B) The decimal point:
Now that we have a bit of an insight into number systems in general, let us come back to the comfort of our familiar "decimal system" and understand an entity known as the "decimal point". To understand the decimal point, we have to briefly review a mixed number such as 8**5/11** ("8 and five-elevenths"). This is a combination of a whole number 8 plus a proper fraction 5/11 ("five-elevenths"). We have seen before how this number can be verbalized either as "8 and five-elevenths" or as "8 plus five-elevenths". The bottom line is that 8**5/11** ("8 and five-elevenths") is a combination of a whole number 8 plus a proper fraction 5/11 ("five-elevenths").

Now look at the following number: 8**.**45   The point shown between the 8 and the 45 is called a "decimal point". This number (8**.**45) has to be verbalized as "8 point four five" and not as "8 point forty-five". (The reason is explained later.)

Just as in the case of the mixed number 8**5/11** ("8 and five-elevenths"), the decimal number 8**.**45 ("8 point four five") is also a combination of a whole number 8 plus a proper fraction shown in decimal format as **.**45 ("point four five"). The only difference between 8**5/11** and 8**.**45 is that 8**5/11** is shown in "mixed number" format (whole number + proper fraction) while 8**.**45 is shown in "decimal format" (whole number + proper fraction shown as a decimal). In fact, the two numbers are the same. (The conversion of one format to another is covered later.)

(.....more details in the original.....)

(66) Writing out hundreds, thousands, millions, billions, etc., for decimal numbers:
I want to revisit the writing out of large numbers, something we did in section # 15. You write out a large number in words, then have your child read the sentence, and write out the number. The only difference here is that the numbers will now contain a decimal.

The process for writing out large numbers that are in decimal format is very similar to what we have already learned.

(.....more details in the original.....)

(69) Percentage:
A good understanding of "percentages" is very important for every child.

"Per" means "for every"; "cent" (from the Latin "centum") means "hundred".
So: "percent" or "percentage" means "for every hundred".
When you use the word "percent", you are making a comparison of a number with the number 100. The percent symbol is **%**. So 28**%** is to be verbalized as "28 percent", and "28 percent" is written as 28**%**.

If I say that "28**%** of the students in this college are Asian", it means that "28 for every 100" students are Asian. Stated slightly differently, I can say that "28 out of every 100" students are Asian. If I receive 80**%** marks in a test, then I have received "80 out of 100" (possible) marks in the test.

(.....more details in the original.....)

(79) Rounding:
This is yet another topic that your child won't be able to escape in school. It could prove a tiny bit tricky, so read this closely.

"Rounding" a number means "approximating" the number. The "rounded" answer must remain very close to the original. As a quick example: We can all agree that 57 is closer to 60 than it is to 50. So, if I was asked to "round off 57 to the nearest 'tens' position", my answer would be 60. Note that 60 is the approximation. The original number is 57. The approximated number must end up close to the original, or something is wrong with your "rounding".

Before we look at the mechanics of rounding off any number to any "position", let us first take a quick review of digit "positions".

(.....more details in the original.....)

(80) Scientific notation:
So far, we have seen numbers written as whole numbers, decimals, fractions, or percentages. Numbers that are in the form of whole numbers (which have an implied decimal point after the number), or decimal numbers are all numbers written in "standard notation". For example: 12345; 1.2345; 0.0345; 287.99; etc., etc.

So even if you had to write very large numbers or very small numbers, you would normally write these in standard notation. For example, 6,000,000,000 (6 billion) is a very large number and 0.0000000006 (six ten-billionths) is a very small number, both written in "standard notation". There is another way of expressing very large or very small numbers.

Scientific Notation (I'll call it "SN", for short.....) is a format for expressing very large or very small numbers. *When used in conjunction with "Rounding", SN allows us to quickly appreciate the magnitude of very large or very small numbers*. This will be a good challenging mental exercise for your child.

(.....more details in the original.....)

(81) Permutations and combinations:
These concepts are good mental exercise for cognitive reasoning. In a nutshell, permutations and combinations are both about "physical arrangements" of items or events. Here is the basic difference between arrangements that are permutations and arrangements that are combinations.

*P*ermutations (with a "*P*") are concerned with *P*ositions (with a "*P*"). Combinations are not concerned with positions.

As a quick example for now, you can permute (arrange with positioning) 3 items named A,B,C as follows: abc, acb, bac, bca, cab, cba, and these are all considered unique and distinct permutations. Because permutations are concerned with "position", it follows that abc, acb, bac, bca, etc. are all considered to be different permutations.

However, you can only combine A,B,C in just one way, regardless of whether you call it abc, acb, bac, bca, etc. This is because abc, acb, bac, bca, etc., are considered one and the same combination. Combinations are not concerned with "position". Think of combinations as mixtures. If you have a mixture of salt and sugar, it hardly matters to you how the salt and sugar particles are aligned with one another. They are still all in the same mixture. That's how it is with combinations.

(.....more details in the original.....)

(82) The "BALANCE" concept in math - crucial !:

In attempting to teach my grandson how to quickly understand and solve "word problems", I coined the term "the balance concept in math". This concept, if properly utilized, makes short work of solving word problems, because most word problems fall under the "balance" concept, as explained in detail in this section. Have your child concentrate on this section. It will help make math life quite a bit easier. The balance concept can be applied in a variety of ways, and I will explain these as we move along.

(82-A) The basics of the "Balance" concept:

Problem 1: 4 books cost $12; how much would I pay for 16 books?
or: How much would I pay for 16 books if 4 books cost $12 ?
or: Calculate the price of 16 books at the rate of 4 books for $12, etc.

Regardless of how the problem is worded, there are clearly 2 sections in this problem:
(1) the data (or "balance") section, that says that "4 books cost $12"; that's given to us; there is no question posed there;
(2) the problem section, which asks how much I would pay for 16 books; this is the portion to be solved.

Let's concentrate on the data or balance section, first:
"4 books cost $12". This is the data section. I also call it the "balance" section, because in this problem, 4 books balance $12. Therefore, in this problem, 4 balances 12.

Note very carefully that I am not saying that "4 is equal to 12". Clearly, 4 is not equal to 12. I am only saying that as far as this problem is concerned, because the data says that "4 books cost $12", it would be quite correct and logical for me to say that "in this problem, 4 balances 12". In this problem, the 4 is balanced by the 12.

(.....more details in the original.....)

(82-B) "Balances" as fractions - very useful !:

So far, we have been showing "balances" as follows:

(a)	9 balances 72:	(b)	4 balances 20
so:	11 balances 88	so:	5 balances 25

regardless of what the "units" might be.

Now, let me show you another way of working with the balance concept in math. This method will greatly enhance your problem-solving ability. Study this section carefully.

Let's see the example of: 9 items cost $72

I can write this as: 9 balances 72 and I would be quite correct.

However, it is better to write balances as fractions! Note this explanation carefully!!

"9 items balance $72" is better written as:

9 items		$72
----------	or	--------
$72		9 items

9 items / $72 can be interpreted as: "9 items purchased, for every $72 spent" or "9 items for every $72"

$72 / 9 items can be interpreted as: "$72 spent, for every 9 items purchased" or "$72 for every 9 items"

(.....more details in the original.....)

(85) Areas and Volumes:

Every child needs to understand the basic concepts of "areas" and "volumes".

(.....more details in the original.....)

(87) Basic Algebra:

Algebra is a branch of math that is useful in every branch of science and mathematics, and it is imperative that every child should have some background in basic Algebra.

The (Anglicized) word "Algebra" comes from a Latin variant of an Arabic phrase "Al Jabr", which translates to "the transposition". We will soon see (in the next section) what "the transposition" is all about, but for now I want to summarize the origin of the word "Algebra". So, the science (or the Math) of Algebra ("Al Jabr") is a contribution of the ancient Arabs.

Algebra works with numbers and symbols to express mathematical ideas. The numbers are called "constants", because their values are fixed. For example, 9 is 9, however you look at it. Its meaning and value are fixed, or constant. It's the same for any other number, positive or negative. Symbols, on the other hand, are unknown quantities that need to be resolved, and are considered "variables" or "unknowns" until resolved. Algebra solves for the symbols, thus bringing value and meaning to the symbols. Before we can embark on the study of basic Algebra, we have to learn some rules associated with expressing numbers and symbols, so that you can understand someone else's Algebra, and, more importantly, everyone can understand your Algebra. We will use numbers (constants) where appropriate, and we will use "x", "y", z" to denote variables where appropriate. You can really use any letter of the alphabet to represent "the unknown or the variable", but traditionally, "x", "y" and "z" have been favorite unknowns in mathematics, and mathematics honors tradition. So for the most part we will stay with x, y, and z to represent our unknown quantities in Algebra.

This brings up one small issue for me, in the writing of the book. If I use the letter "x" to represent a "variable", then I cannot use the letter x to represent the "multiplication" function as I have been doing so far. So from now on, I will use the ***** (*asterisk*) symbol to represent the multiplication function. Incidentally, when programming computers, you have to use the ***** symbol in computer programs to show the multiplication function. But that's another story.....

(.....more details in the original.....)

(88) Basic Geometry:

The word "Geometry" derives from the Greek words "geo" (meaning Earth) and "metria" (meaning "measurement"). The Greek mathematician "Euclid" is recognized as the Father of Geometry for his contribution to this branch of math. In a nutshell, Geometry is concerned with the size, shape and inter-relationships of 2-dimensional (2-D) and 3-dimensional (3-D) objects and figures. The study of 2-D objects and figures is called "Plane" Geometry, and the study of 3-D objects and figures is called "Solid" Geometry.

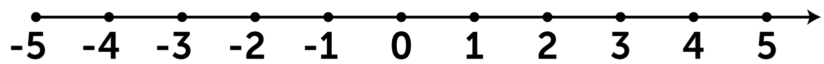
In this section on Geometry, we will concern ourselves with a small subset of "Plane" Geometry. There is also a branch of "Plane" Geometry called "Coordinate" Geometry, which studies relationships between geometrical shapes and Algebraic values and Algebraic expressions, and we will see some of that in the next section. The sole purpose of introducing these topics to your child is to get him to think logically and to reason.

(.....more details in the original.....)

<u>(89) Basic coordinate Geometry</u>:

"Coordinate Geometry" is the math that studies <u>relationships between Geometrical shapes and Algebraic values and expressions</u>. It's a marriage of Algebra and Geometry.

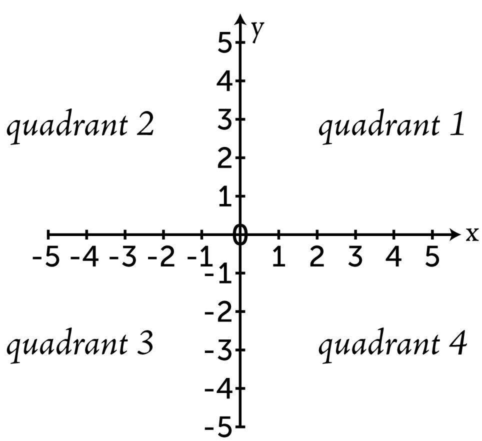
Let us take a quick look at the "number-axis" again:



(Fig. 5 - again)

The number-axis helped us to understand that all numbers are merely points on a line extending into plus-infinity on the right and minus-infinity on the left. We learned the concepts of: the origin (zero); positive and negative numbers; larger and smaller numbers; ascending and descending numbers; basic addition and subtraction concepts; etc.

We are now going to take a big leap forward in conceptualization. We will add another number-axis to our horizontal number-axis, this one moving up and down in the <u>vertical</u> direction. See figure 52:



(Fig. 52)

(.....more details in the original.....)

(90) Basic important concepts in general science:
Every young child should be aware of the following important scientific concepts.

(90-A) Law of conservation of mass and energy:
This law neatly sums up the fact that "you cannot get something for nothing".
The Law of Conservation of Mass and Energy provides a very important concept that everyone needs to understand and appreciate. Have your child repeat this law to you several times. The actual statement of this law may vary slightly from one science text-book to another.

"The Law of Conservation of Mass and Energy states that the total amount of Mass and Energy in the universe is constant. Mass and Energy can be neither created nor destroyed. Mass and Energy can only be converted from one form into another."

In other words, there is no such thing as a "freebee", in the physical universe.

Before we take some examples clarifying the above law, let me briefly describe the meaning of the word "Friction".

(.....more details in the original.....)

(90-B) Einstein's Equation:
$E = MC^2$ This is probably the most famous equation in the world.
This equation unites "mass" and "energy". The equation states:

"E" is the energy (in ergs or Joules) liberated when a mass of "M" (grams or Kilograms) disappears into pure energy. "C" is the speed of light **squared** (in centimeters per second or Meters per second).

(.....more details in the original.....)

(90-E) Electromagnetic induction:
There should be no doubt in anyone's mind that without the energy form known as "electricity", this world would be thrown back into the stone-age. We cannot live the way we are accustomed to, without the practical application of electrical energy. Homes (and just about everything in them), factories, offices, banks, computer installations, hospitals, schools, restaurants, planes, trains, automobiles and anything else you can think of all depend on electrical power to keep them going. If you have ever had a power outage affect your home, you know exactly how frustrating and upsetting that can be.

Our practical use of electricity would not have been possible without the contributions of an English scientist named Michael Faraday who discovered the phenomenon called "electromagnetic induction". I think that a small introduction to this topic would be very beneficial to every child.

(.....more details in the original.....)

(90-H) What is color **?**:

I plugged in this small section as a last-minute thought. It was triggered by my reading about a competition in which entrants were asked to define the words "color" and "flame".

(.....more details in the original.....)

Now, we can "see" things only if the following conditions are met:

1. We have eyes that are not damaged, and are functioning well.

2. Light from an object must reach the eye, in order that we may see the object. Note that we cannot see light itself. We can only see the effect of light on objects around us. So light must fall on an object, then bounce off the object and reach our eyes, for us to be able to see the object.

3. The light from the object must be scattered light, for us to see the object. It is impossible for us to see perfectly-transparent or perfectly-reflecting objects.
For example: we cannot see air, which is perfectly transparent, but we can see smoke and dust in the air because they scatter light. We cannot see the surface of a perfectly-polished mirror, but we can see images of objects reflected in that mirror. However, if a bit of dust settles on the surface of this perfectly-polished mirror, then the surface becomes visible. Bottom line: it is scattered light that allows us to see objects.

4. The object must sharply contrast with the surroundings, for us to see the object. For example, a black cloth spread across a black background is impossible to see, and is the cornerstone of many tricks of "magic". A green parrot in a tree in summer is impossible to spot even though you can clearly hear the bird, because it is "camouflaged" in the leaves. All kinds of animals use camouflage for protection, as do military personnel.

5. How we see color (our perception of what is color) depends very much on the kind of light reaching the eyes. This will become clear very shortly.

(.....more details in the original.....)

<u>What is "Light"</u>?

The concept of "light" has puzzled scientists for many years, and over the centuries many theories have been floated as to what constitutes light. <u>Light is **something** that helps us see</u>. Keeping this down to a very rudimentary discussion, let me say that there are basically two popular theories of light, one which states that light is <u>a type of electromagnetic radiation</u> that travels in the form of "<u>waves</u>", and the other which states that light is <u>a type of electromagnetic radiation</u> that travels in the form of "<u>particles</u>" called "photons". The one point of agreement is that "<u>light is a type of electromagnetic radiation</u>".

(.....more details in the original.....)

<u>Why do objects appear colored</u>?

"White light" from the sun is made up of a very wide <u>spectrum</u> of colors (frequencies). This spectrum of colors from sunlight can be collated into 7 groups of colors, which we refer to generally as "violet, indigo, blue, green, yellow, orange, and red". Note that these 7 groups of colors are not "optically pure". Each of these 7 groups of colors comes in many different closely-related frequencies, which is why we say that light from the sun is made up of a very broad <u>spectrum</u> of frequencies (colors). Collectively, this spectrum of colors from sunlight appears to us as "<u>white light</u>".

(.....more details in the original.....)

<u>(90-I) What is a flame</u> ?:

I plugged in this small section as a last-minute thought. It was triggered by my reading about a competition in which entrants were asked to define the words "flame" and "color". Some of the definitions of "flame" were quite outlandish and verbose. So I decided to plug in my definition of the word "flame", something that I had learned in the 8th grade back in Bombay (Mumbai), India.

"<u>A flame is a **gas** which is so hot that it gives out light</u>." (That's it - short and sweet.)

(.....more details in the original.....)